Whirlygigs for Sale!
Rotating Two-Dimensional Figures through Space

The ChocoWorld Candy Company is going to enter a candy competition in which they will make a structure entirely out of chocolate. They are going to build a fairytale castle using several different molds, and they need to make the molds using a drill bit that will create the shape they are striving for.

1. The castle will need several turrets, which are made by pouring chocolate into a mold that will form a cone.
   a. Which of the figures shown is a cone?

   ![Figures](image)  
   - Fig. 1  
   - Fig. 2  
   - Fig. 3  
   - Fig. 4

   b. Which of the drill bits shown will form a cone after being rotated in a plastic molding compound?

   ![Drill Bits](image)  
   - A  
   - B  
   - C  
   - D
c. What is the shape on the drill bit that forms the cone in the mold as it is being rotated?

d. If the triangle on the drill bit is 2 inches wide and 1 inch tall, what will the dimensions of the cone be that is formed by the rotation of the bit?

2. The castle that the company is making is going to have long circular columns in the front.
   a. What type of solid mold is needed to create circular columns?
   
   b. Which of the drill bits shown will form the mold needed after being rotated in the plastic molding compound?

   ![Drill Bits]

   A  B  C  D

   c. What is the shape of the drill bit that will create the column mold?

   d. If the width of the shape on the end of the drill bit being rotated is 3 inches, what is the radius of the base of the cylinder going to be?
To complete the castle, the company is going to create small cannonballs for the cannon that will be situated on the roof of the castle.

a. What type of solid mold is needed to make cannonballs?

b. Which of the drill bits shown will form the mold needed if the cannonballs will have a radius of 0.25 inch?

- A
- B
- C
- D

c. What is the shape of the drill bit that will create the cannonball mold?
Cakes and Pancakes
Translating and Stacking Two-Dimensional Figures

Theodore is starting a new company that will manufacture food storage containers. He asks his engineers to design several different containers based on which type of containers will sell the best.

1. The end of one container is a rectangle

   ![Rectangle Diagram]

   a. Translate the rectangle in a diagonal direction to create a second rectangle.

   b. Use dashed line segments to connect each pair of corresponding vertices in the rectangles.

   c. What do you notice about the relationship among the line segments in your drawing?

   d. What is the name of the solid formed by this translation?
2. The base of one container is a triangle.

   a. Translate the triangle in a diagonal direction to create a second triangle.

   b. Use dashed line segments to connect each pair of corresponding vertices in the triangles.

   c. What do you notice about the relationship among the line segments in your drawing?

   d. What is the name of the solid formed by this translation?
3. Theodore gets a contract from a restaurant to make containers for their soup. The bottom of the container is a disc. An oval is drawn to represent what the base of the container might look like.

a. Translate the oval in a diagonal direction to create a second oval.

b. Use dashed line segments to connect the tops and the bottoms of the ovals.

c. What do you notice about the relationship between the line segments in your drawing?

d. What is the name of the solid formed by this translation?
The Harrington Heights Middle School is getting ready for its spring musical. The students in the art classes are using donated cardboard boxes to make the props.

4. One of the props for the show needs to be a suitcase. The directors want its weight to be light because it is used in one scene to playfully hit another actor. The art students decide to make the suitcase shape by stacking rectangles of the same shape and size on top of each other. Each cardboard cutout is 48 inches long by 30 inches wide.

a. The students are going to stack 4 rectangular cutouts on top of each other. What is the name of the solid formed by this stack of rectangles?

b. If the thickness of each cardboard rectangle is 0.25 inch, what is the thickness of the suitcase?

c. Relate the dimensions of the suitcase formed to the dimensions of the rectangles.

d. The directors of the show have decided that they would like the suitcase to have a greater thickness than 1 inch. They would like it to have a thickness of 3 inches. How many rectangles do the students need to stack?
5. The art students need to work on making a stop sign for the show using stacks of figures that have similar shapes and sizes.
   a. What type of shape will they need to cut out from the cardboard for their stack?
   
   b. What is the name of the solid that will be formed by this stack?
   
   c. The cardboard the students use for the stop sign is 0.125 inch thick. If the directors want the stop sign to be 1 inch thick, how many stop signs will the students need to stack?

6. One of the scenes of the musical involves a scene on city streets. The director would like to have several traffic cones set up for the scene.
   a. The students need to build up each cone using stacks of shapes. What type of shapes will they use? What should the size of the shapes be?
   
   b. What is the relationship among the discs getting stacked?
   
   c. The cardboard they are using for the cone is 0.5 inch thick. The directors want the traffic cones to be 14 inches tall. How many discs will the students need to stack for each cone?
Cavalieri's Principles
Application of Cavalieri’s Principles

1. Divide the figure shown into approximately 10 rectangles. What is the length, the height, and the area of each rectangle?

2. What is the approximate area of the irregularly shaped figure?

3. If this irregularly shaped figure were divided into 1000 congruent rectangles, what would be the approximate area of the figure?

4. If this irregularly shaped figure were divided into \( n \) congruent rectangles, what would be the approximate area of the figure?
The Leaning Tower of Pisa in Italy is about 180 feet tall from the top of the tower vertically to the ground. It has a diameter of approximately 51 feet.

5. Determine the approximate volume of the tower. Explain your reasoning.
Spin to Win
Volume of Cones and Pyramids

1. Joel owns a frozen yogurt and fruit smoothie shop. He just placed an order for three different sizes of cones. He needs to determine how much to charge for each cone and decides that knowing the volume of each might help him make his decision.

   a. Which cone do you think has the greatest volume? Explain your reasoning.

   b. Identify the radius, the diameter, and the height of cone 1. How did you determine the radius of the cone?
c. Calculate the volume of cone 1. Show your work. Round your answer to the nearest hundredth.

d. Identify the radius, the diameter, and the height of cone 2. How did you determine the diameter of the cone?

e. Calculate the volume of Cone 2. Show your work. Round to the nearest hundredth.
f. Identify the radius, the diameter, and the height of cone 3. How did you determine the radius of the cone?

g. Calculate the volume of Cone 3. Show your work. Round to the nearest hundredth.

h. Determine which size cone Joel should charge the most for and the least for. Explain your reasoning.

i. Compare the volumes of all three cones.
2. Pyramid tents were popular for a time during the 19th century. Although their popularity declined during the 20th century, they have recently begun to regain popularity again. The design is ideal for shaping canvas, and it only requires one pole and some stakes to secure it. Joe wants to make a right square pyramid tent and is considering two different sizes. He will either make one with a base that is 10 feet by 10 feet and has a height of 12 feet, or he will make one with a base that is 12 feet by 12 feet and has a height of 8 feet.

a. Sketch the two pyramid designs Joe is considering and label them with the given measurements.

b. How can you determine which pyramid tent will have the most interior space?

c. Calculate the volume of each proposed pyramid tent. Show your work.

d. Which tent would you recommend Joe make? Explain your reasoning.
Spheres à la Archimedes
Volume of a Sphere

Calculate the volume of each sphere. Use 3.14 for \( \pi \) and round to the nearest tenth, if necessary.

1. 21 mm

2. 45 ft

3. A can holds 3 tennis balls as shown in the figure. The radius of each tennis ball is 3 centimeters.
   a. What is the volume of a single tennis ball?
   b. What is the total volume all 3 tennis balls take up?
c. Can you determine the height of the can? Explain your reasoning.

d. What is the volume of the can? Use 3.14 for $\pi$.

e. What is the volume of the can not taken up by the tennis balls?
LESSON 4.6 Assignment

Name ___________________________________________ Date _____________

Surface Area
Total and Lateral Surface Area

1. Julie plans to redecorate her bedroom. A model of her bedroom is shown.
   a. Determine the number of square feet of flooring she will need to purchase.

   b. How many square yards of carpet should Julie order?

   c. Julie would like to paint the walls in her bedroom. Determine the square footage of the walls.

   d. If a can of paint will cover 300 square feet, how many cans of paint will Julie need to purchase?
2. Determine the minimum amount of wrapping paper Juan will need to wrap a soccer ball that comes in a pentagonal prism box. The height of the box is 12 inches, each side of the regular pentagon is 8 inches and the apothem of the pentagon is 5 inches.

   a. Determine the lateral surface area.

   b. Determine the base area.

   c. Determine the total surface area.
3. Tim is building a teepee style tent that is shaped like a right hexagonal pyramid. In order for Tim to stand up in the center, the height should be 6 feet and the slant height should be 10 feet. Each side of the regular hexagon is 5 feet.

a. Determine how many square feet of canvas Tim will need to purchase to cover the sides.

b. Tim needs to buy support poles for the sides. How many linear feet will he need?

c. Tim would like to cover the floor of the tent with a rug. The apothem of the hexagon is not given, but the area can still be determined. Determine the area of the rug and explain your reasoning.
LEsson 4.7 Assignment

Name ___________________________ Date _____________

Turn Up the . . .
Applying Surface Area and Volume Formulas

1. The Luxor Hotel in Las Vegas is a replica of the Pyramid of Khafre at Giza, one of the seven wonders of the world. The Luxor’s base is a square with a side length of 646 feet, and it is 350 feet tall.
   a. What is the volume of the Luxor Hotel?

   b. The Pyramid of Khafre has a volume of 2,226,450 cubic meters. Its base is a square with a side length of 215 meters. What is the height of the Pyramid of Khafre?
2. A store sells square pyramid-shaped scented candles. The dimensions of two of the candles are shown.

![Candle A](image1)

Candle A: 6 cm x 6 cm base, height 16 cm

![Candle B](image2)

Candle B: 8 cm x 8 cm base, height 9 cm

a. Calculate the volume of each candle.

b. Both candles are made of wax. Which candle contains more wax? Explain.
c. Suppose the candles have a sparkly coating on the sides. This option will cost 2.5 cents per square centimeter. Determine the lateral surface area of each candle and the additional cost for the sparkly coating.

d. Which candle will cost more? Explain.
3. The manager of Sunnybrooke Farm plans to buy a new grain silo. He needs to choose between two options. Silo A is a cylinder of height 25 feet and a radius of 8 feet. The roof is a hemisphere with the same radius. Silo B is a cylinder of height 20 feet and a radius of 10 feet. The roof is a hemisphere with the same radius. Use 3.14 for $\pi$ and round to the nearest hundredth.

**Silo A**
- Height: 25 ft
- Radius: 8 ft

**Silo B**
- Height: 20 ft
- Radius: 10 ft

a. Determine the volume of each silo.
b. The farm manager will have to paint the outside of whichever silo he decides to purchase. Determine the surface area to be painted. Use 3.14 for $\pi$.

c. The silo needs to be painted every 5 years. Which silo will be more expensive to maintain?
4. A street vendor in London sells fish and chips in a paper cone. The height of the cone is 15 centimeters and it holds approximately 1000 cubic centimeters of food.
   a. Determine the approximate diameter of the cone.
   b. Determine the amount of paper used to make the cone.

5. A funnel that is used to change the oil in a car is in the shape of a cone. The base of the funnel has a circumference of 60 centimeters. The height of the funnel is 25 centimeters. How much oil will this funnel hold?
6. A cylindrical tank has a height of 8 feet and a radius of 3 feet.
   a. If the radius and height are doubled, determine how the volume is affected.
   b. If the radius and height are tripled, determine how the surface area is affected.
c. If the radius and height are each increased by 5 feet, determine the affect on the surface area and volume.
7. As a child, Meredith stored keepsakes in a small wooden chest like the one shown.
   a. Describe the three-dimensional figures that make up the chest.

   b. Meredith decided to paint the outside of the chest. Determine the surface area that needs to be painted.
c. Meredith would like to store a quilt from her grandmother in the chest. The quilt is currently folded and forms a rectangular prism with dimensions of 13 inches, 25 inches, 4 inches. If the quilt is refolded, will it fit in the chest? Will there be any leftover space in the chest? Explain how you determined your answers.
Tree Rings
Cross Sections

Describe the shape of each cross section.

1. 

2. 

3. 

4. 

5. 

6.
7. Sketch two cross sections of a pentagonal prism—one cross section that is parallel to the base and another cross section that is perpendicular to the base.

8. Sketch two cross sections of a cone—one cross section that is parallel to the base and another cross section that is perpendicular to the base.

9. A solid’s cross section parallel to the base is an octagon. A cross section of the solid perpendicular to the base is a triangle. Identify the solid.

10. A solid’s cross section parallel to the base is a triangle. A cross section of the solid perpendicular to the base is a rectangle. Identify the solid.
Two Dimensions Meet Three Dimensions
Diagonals in Three Dimensions

1. What is the length of a three-dimensional diagonal of the rectangular prism?

   ![Diagram of a rectangular prism with dimensions 11 cm, 6 cm, and 4 cm]

   - 4 cm
   - 6 cm
   - 11 cm

2. What is the length of a three-dimensional diagonal of the rectangular prism?

   ![Diagram of a rectangular prism with dimensions 18 in, 15 in, and 12 in]

   - 12 in.
   - 15 in.
   - 18 in.
3. A rectangular box has a length of 6 feet and a width of 2 feet. The length of a three-dimensional diagonal of the box is 7 feet. What is the height of the box?

4. The length of the diagonal across the front of a rectangular box is 20 inches, and the length of the diagonal across the side of the box is 15 inches. The length of a three-dimensional diagonal of the box is 23 inches. What is the length of a three-dimensional diagonal of the box?

5. Pablo is packing for a business trip. He is almost finished packing when he realizes that he forgot to pack his umbrella. Before Pablo takes the time to repack his suitcase, he wants to know if the umbrella will fit in the suitcase. His suitcase is in the shape of a rectangular prism and has a length of 2 feet, a width of 1.5 feet, and a height of 0.75 foot. The umbrella is 30 inches long. Will the umbrella fit in Pablo’s suitcase? Explain your reasoning.