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### Chapter 3 Overview

In this chapter, students investigate strategies for determining the perimeters and areas of rectangles, triangles, non-rectangular parallelograms, trapezoids, and composite plane figures on the coordinate plane. Students also explore the effects of proportional and non-proportional changes to the dimensions of a plane figure on its perimeter and area.

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Chapter 3  Perimeter and Area of Geometric Figures on the Coordinate Plane
## Skills Practice Correlation for Chapter 3

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<td>3.5</td>
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<td>Determine the perimeter and area of composite figures on the coordinate plane using transformations</td>
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Using Transformations to Determine Area

**LEARNING GOALS**

In this lesson, you will:

- Determine the areas of squares on a coordinate plane.
- Connect transformations of geometric figures with number sense and operations.
- Determine perimeters and areas of rectangles using transformations.
- Determine and describe how proportional and non-proportional changes in the linear dimensions of a rectangle affect its perimeter and area.

**ESSENTIAL IDEAS**

- Rigid motion is used to change the position of rectangles and squares on the coordinate plane.
- Rigid motion is used to determine the area of rectangles.
- The Distance Formula is used to determine the area of rectangles and squares on the coordinate plane.
- When the dimensions of a plane figure change proportionally by a factor of $k$, its perimeter changes by a factor of $k$, and its area changes by a factor of $k^2$.
- When the dimensions of a plane figure change non-proportionally, its perimeter and area increase or decrease non-proportionally.

**TEXAS ESSENTIAL KNOWLEDGE AND SKILLS FOR MATHEMATICS**

(3) Coordinate and transformational geometry.

The student uses the process skills to generate and describe rigid transformations (translation, reflection, and rotation) and non-rigid transformations (dilations that preserve similarity and reductions and enlargements that do not preserve similarity). The student is expected to:

- (B) determine the image or pre-image of a given two-dimensional figure under a composition of rigid transformations, a composition of non-rigid transformations, and a composition of both, including dilations where the center can be any point in the plane.
Overview
Rigid motion is used to explore the perimeter and area of rectangles and squares on the coordinate plane in this lesson. Given graphs of differently oriented rectangles and squares, students will compute the perimeter and area using necessary formulas. Through the use of translations, the rectangles and squares are positioned at the origin to ease arithmetic computation. Students also investigate how proportional and non-proportional changes to the linear dimensions of a plane figure affect its perimeter and area.
Warm Up

Four points and their coordinates are given.

A (5, 8)  D (12, 8)

B (5, -1)  C (12, -1)

1. Do you think quadrilateral $ABCD$ is a rectangle? Explain your reasoning.
   
   Yes. Quadrilateral $ABCD$ appears to be a rectangle.
   
   The consecutive sides appear to be perpendicular to each other and the opposite sides appear to be parallel to each other.

2. Compute the perimeter of quadrilateral $ABCD$.

   $9 + 9 + 7 + 7 = 32$
   
   The perimeter of quadrilateral $ABCD$ is 32 units.

3. Compute the area of quadrilateral $ABCD$.

   $A = bh$
   
   $= (9)(7) = 63$
   
   The area of quadrilateral $ABCD$ is 63 square units.
You've probably been in a restaurant or another public building and seen a sign like this:

**MAXIMUM OCCUPANCY**

480

What does this mean? It means that the maximum number of people that can be in that space cannot—by law—be more than 480.

Why does this matter? Well, occupancy laws are often determined by the fire marshal of a town or city. If too many people are in a space when an emergency occurs, then getting out would be extremely difficult or impossible with everyone rushing for the exits. So, occupancy laws are there to protect people in case of emergencies like fires.

Of course, the area of a space is considered when determining maximum occupancy. Can you think of other factors that should be considered?
Problem 1
A scenario is used containing a quadrilateral on the coordinate plane. The coordinates for each of the four vertices are provided. Students list the properties of squares and non-square rectangles and use the Distance Formula and the slope formula to determine if the quadrilateral is a square or a rectangle. They also use a rule describing the maximum occupancy of a room to determine how many people can safely fit into a specified area.

Grouping
Have students complete Questions 1 through 7 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Questions 1 through 7
• Are there any assumptions you can make by only an observation in geometry?
• Can you assume a vertical pair of angles by observation alone?
• Can you assume a perpendicular line relationship by observation alone?
• Can you assume a parallel line relationship by observation alone?
• Can you assume an intersecting line relationship by observation alone?
• What is the difference between a square and a rectangle?
• What properties do a square and a rectangle have in common?
• Why is the rule of thumb important?
• How would you describe the approximate size of 36 square feet?
• What formulas are used to determine if the quadrilateral is a square or a rectangle?
• Is the location too large or too small for Marissa’s party?

Problem 1  Pomp and Circumstance
Marissa is throwing a party for her graduation and wants to invite all of her friends and their families. Consider the space defined by quadrilateral ABCD. Each of the four corners of the space is labeled with coordinates, measured in feet, and defines the dimensions of the room that Marissa’s little brother says the party should be held.

1. Marissa’s mom says that the room is obviously a square or a rectangle, so if you can figure out the length of one or two of the sides, then you can easily determine the area. Marissa tells her mother that you can’t just assume that a shape is a square or a rectangle because it looks like one. Who is correct and why?

Marissa is correct. Non-square rectangles and squares have specific geometric properties. A figure can look like a non-square rectangle or a square, but if it doesn’t have the specific properties, it isn’t one.
2. List the properties of each shape.
   
   a. squares
   A square is a quadrilateral—it is a polygon with 4 sides. It has 4 congruent sides and 4 right angles.
   
   b. non-square rectangles
   A rectangle is a quadrilateral—it is a polygon with 4 sides. It has 2 pairs of parallel sides and 4 right angles. The parallel sides are congruent.

3. How can you use the properties you listed in Question 2 to determine whether the room is a square or a non-square rectangle?
   I need to show that it has 4 right angles. That would make it a rectangle. To determine whether or not it is a square, I need to show that the side lengths of the room are all equal and that it has 4 right angles. That means that I need to use the Distance Formula to determine the lengths of the sides. Then, I need to show that all of the sides are perpendicular to each other. I can do this by comparing the slopes of the line segments using the coordinates of the points of the vertices.

4. A rule of thumb for determining the maximum occupancy of a room is that each person in the room is given 36 square feet of space.
   Predict the maximum occupancy of the room Marissa wants to rent. Describe the information you need and the strategies you could use to improve your prediction.
   Answers will vary.
   Student responses could include:
   I need to know the area of the room in square feet. I can then divide that by 36 to determine the maximum occupancy of the room.
   To determine the area, I need to calculate at least 2 side lengths. I also need to know if the quadrilateral is a rectangle or not.
5. Determine if quadrilateral $ABCD$ is a square or a rectangle. Show your work.

For $AB$:

$$AB = \sqrt{(2 - (-3))^2 + (15 - 3)^2}$$

$$= \sqrt{5^2 + 12^2}$$

$$= \sqrt{25 + 144}$$

$$= \sqrt{169}$$

$$= 13$$

For $CD$:

$$CD = \sqrt{(14 - 9)^2 + (10 - (-3))^2}$$

$$= \sqrt{(5)^2 + (12)^2}$$

$$= \sqrt{25 + 144}$$

$$= \sqrt{169}$$

$$= 13$$

For $BC$:

$$BC = \sqrt{(-3 - 9)^2 + (3 - (-2))^2}$$

$$= \sqrt{(-12)^2 + (5)^2}$$

$$= \sqrt{144 + 25}$$

$$= \sqrt{169}$$

$$= 13$$

For $AD$:

$$AD = \sqrt{(14 - 2)^2 + (10 - (-15))^2}$$

$$= \sqrt{(12)^2 + (25)^2}$$

$$= \sqrt{144 + 625}$$

$$= \sqrt{769}$$

Slope of $AB$: $\frac{15 - 3}{2 - (-3)} = \frac{12}{5}$

Slope of $CD$: $\frac{-2 - 10}{9 - 14} = \frac{-12}{-5} = \frac{12}{5}$

Slope of $BC$: $\frac{3 - (-2)}{-3 - 9} = \frac{5}{-12} = -\frac{5}{12}$

Slope of $AD$: $\frac{10 - 15}{14 - 2} = \frac{-5}{12}$

All 4 sides of the quadrilateral are congruent. The slopes of the adjacent sides are all negative reciprocals, meaning that they are perpendicular and form right angles. So, quadrilateral $ABCD$ is a square.

6. Use the rule of thumb from Question 4 to determine the maximum number of people that Marissa can invite to her party.

The area of the space is only $13^2$, or 169, square feet. Divide by 36 to estimate maximum occupancy: $\frac{169}{36} < 4.69$.

Marissa can invite only 3 people to the party, given the size of the room.

7. Do you think this location is reasonable for Marissa’s graduation party? Why or why not?

This location is not reasonable for Marissa’s graduation party because it is too small. Marissa will have more than 3 people attending her party.
Problem 2
A figure divided into two quadrilaterals is drawn on a coordinate plane. The coordinates of all vertices of both quadrilaterals are provided. Students describe a strategy that can be used to determine the total area of the figure. Students also determine and apply a transformation to decide if the two quadrilaterals are congruent.

Grouping
Have students complete Questions 1 through 6 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Question 1
- What shape do the quadrilaterals appear to be?
- How would you determine if the quadrilaterals are rectangles?
- How would you determine if the quadrilaterals are squares?

PROBLEM 2 Maximum Efficiency

In addition to formulas and properties, you can also use transformations to help make your problem solving more efficient.

The figure shown on the coordinate plane is composed of two quadrilaterals, FGHJ and KLMN.

1. Describe the information you need and the strategies you can use to determine the total area of the figure. The figure is made up of two quadrilaterals, which appear to be rectangles. I need to show that all 4 angles of each quadrilateral are right angles. If I can show that the figures are rectangles, then I can multiply the length and the width of each quadrilateral to determine its area. Then, I can add the two areas together to determine the total area for the figure. You don’t need to calculate anything yet! Just determine what you need and think about a strategy.
Guiding Questions for Share Phase, Questions 2 through 6

- What is one way Colby could determine if the two quadrilaterals are congruent?
- If the quadrilateral maps onto itself through one or more translations, is that enough to determine the quadrilaterals are congruent?
- Is a vertical, horizontal, or diagonal translation helpful in determining if the quadrilaterals are congruent?
- When a vertex is on an axis or at the origin, which coordinate is always used?

2. Colby says that the two quadrilaterals are congruent. He says that knowing this can help him determine the area of the figure more efficiently. Is Colby correct? Explain your reasoning.

Colby might be correct that the quadrilaterals are congruent, but he hasn’t proved that yet. If the two quadrilaterals are congruent, however, he can determine the area more efficiently, because he would only need to determine the area of one quadrilateral and then double each value to get the total area.

3. Describe a transformation you can use to determine whether the two quadrilaterals are congruent. Explain why this transformation can prove congruency.

I can translate each of the vertices of one of the quadrilaterals. If the translations are the same and result in the coordinates of the vertices of the other quadrilateral, then I know that they are congruent. A transformation such as a translation, a reflection, or a rotation of a geometric figure preserves its shape and size. The pre-image and the image are congruent.

4. Apply the transformation you described in Question 3 to determine if the quadrilaterals are congruent. Show your work and explain your reasoning.

Each of the vertices of quadrilateral $FGHJ$ has the same $y$-coordinate as the corresponding vertex of quadrilateral $KLMN$. Also, each of the $x$-coordinates of the vertices of quadrilateral $FGHJ$ is translated to the right 135 units to result in the corresponding $x$-coordinate of quadrilateral $KLMN$. Quadrilateral $KLMN$ is a horizontal translation of quadrilateral $FGHJ$. Therefore, quadrilaterals $FGHJ$ and $KLMN$ are congruent.
5. Tomas had an idea for solving the problem even more efficiently.

**Tomas**

When a polygon has vertices that are on the x- or y-axis or are at the origin, it is a little easier to use the Distance Formula, because one or more of the coordinates are 0.

Explain why Tomas is correct.

The Distance Formula is \( \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \). If any or both of the points \((x_1, y_1)\) and \((x_2, y_2)\) are on an axis or are at the origin, then one of the coordinates will be 0. This means that I would just have to square the other coordinate to calculate one of the addends under the radical in the Distance Formula.

6. Which quadrilateral would Tomas choose and why? Determine the area of the entire figure.

Tomas would use quadrilateral KLMN, because it has one vertex that is at the origin.

- \(NM = \sqrt{(68.75 - 0)^2 + (67.5 - 0)^2}\)
  \[= \sqrt{(68.75)^2 + (67.5)^2}\]
  \[= \sqrt{4726.56 + 4556.25}\]
  \[= \sqrt{9282.81}\]
  \[\approx 96.35\]
- \(KN = \sqrt{(-135 - 0)^2 + (137.5 - 0)^2}\)
  \[= \sqrt{(-135)^2 + (137.5)^2}\]
  \[= \sqrt{18225 + 18906.25}\]
  \[= \sqrt{37131.25}\]
  \[\approx 192.69\]

The area of quadrilateral KLMN is approximately 96.35 \( \times \) 192.69, or 18,565.68 square units.

So, the area of both quadrilaterals is approximately \(2 \times 18,565.68\), or 37,131.36 square units.
Problem 3

Students are given 3 different rectangles and asked to complete a table to record the changes in the perimeters and areas of the rectangles after a doubling or tripling of each rectangle’s dimensions. Students then answer questions and generalize about the effects on perimeter and area of proportional changes to the dimensions of a rectangle. This activity is repeated with non-proportional changes to the dimensions.

GROUPING

Ask students what they think is meant by a proportional change. Some students may use the term scale factor. Encourage students to ask other students when unfamiliar terms are used. Guide students to conclude that a proportional change is a result of a multiplication. Then ask if this multiplication can only apply to one dimension or if it should apply to two dimensions.

Guiding Questions for Share Phase, Questions 1 and 2

• What happens to the perimeter when the dimensions are doubled? tripled?
• How do these changes relate to the doubling factor (2) and tripling factor (3)?
2. Describe how a proportional change in the linear dimensions of a rectangle affects its perimeter.
   a. What happens to the perimeter of a rectangle when its dimensions increase by a factor of 2?
      When the dimensions of a rectangle increase by a factor of 2, the resulting perimeter is 2 times greater than the original perimeter.

   b. What happens to the perimeter of a rectangle when its dimensions increase by a factor of 3?
      When the dimensions of a rectangle increase by a factor of 3, the resulting perimeter is 3 times greater than the original perimeter.

   c. What would happen to the perimeter of a rectangle when its dimensions increase by a factor of 4?
      When the dimensions of a rectangle increase by a factor of 4, the resulting perimeter will be 4 times greater than the original perimeter.

   d. Describe how you think the perimeter of the resulting rectangle would compare to the perimeter of a 4 \times 10 rectangle if the dimensions of the original rectangle were reduced by a factor of \( \frac{1}{2} \). Then, determine the perimeter of the resulting rectangle.
      The perimeter of the resulting rectangle, 14 inches, would be half of the perimeter of the original rectangle, 28 inches.
      Dimensions of Resulting Rectangle (inches):
      Base = 4 \times \frac{1}{2} = 2
      Height = 10 \times \frac{1}{2} = 5
      Perimeter of Resulting Rectangle (inches):
      2(2 + 5) = 14

   e. In terms of \( k \), can you generalize change in the perimeter of a rectangle with base \( b \) and height \( h \), given that its original dimensions are multiplied by a factor \( k \)?
      The perimeter of a rectangle with base \( b \) and height \( h \) will change by a factor of \( k \), given that its original dimensions are multiplied by a factor \( k \).
3. Describe how a proportional change in the linear dimensions of a rectangle affects its area.
   a. What happens to the area of a rectangle when its dimensions increase by a factor of 2?
      When the dimensions of a rectangle increase by a factor of 2, the resulting area is 4 times greater than the original area.
   b. What happens to the area of a rectangle when its dimensions increase by a factor of 3?
      When the dimensions of a rectangle increase by a factor of 3, the resulting area is 9 times greater than the original area.
   c. What would happen to the area of a rectangle when its dimensions increase by a factor of 4?
      When the dimensions of a rectangle increase by a factor of 4, the resulting area will be 16 times greater than the original area.
   d. Describe how you think the area of the resulting rectangle would compare to the area of a $4 \times 10$ rectangle if the dimensions of the original rectangle were reduced by a factor of $\frac{1}{2}$. Then, determine the area of the resulting rectangle.
      The area of the resulting rectangle, 10 square inches, would be one-fourth of the area of the original rectangle, 40 square inches.
      Dimensions of Resulting Rectangle (inches):
      Base $= 4 \cdot \frac{1}{2} = 2$
      Height $= 10 \cdot \frac{1}{2} = 5$
      Area of Resulting Rectangle (inches):
      $2(5) = 10$
   e. In terms of $k$, can you generalize change in the area of a rectangle with base $b$ and height $h$, given that its original dimensions are multiplied by a factor $k$?
      The area of a rectangle with base $b$ and height $h$ will change by a factor of $k^2$, given that its original dimensions are multiplied by a factor $k$. 

Guiding Questions for Share Phase, Question 3

- What happens to the area when the dimensions are doubled? tripled?
- How do these changes relate to the doubling factor (2) and tripling factor (3)?
Grouping
Have students complete Questions 4 through 7 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Questions 4 through 7

- What happens to the perimeter when the dimensions are doubled? tripled?
- How do these changes relate to the doubling factor (2) and tripling factor (3)?
- What happens to the area when the dimensions are doubled? tripled?
- How do these changes relate to the doubling factor (2) and tripling factor (3)?
- What operation(s) represent(s) a proportional change? a non-proportional change?

Non-proportional change to linear dimensions of a two-dimensional figure involves adding or subtracting from the side lengths.

4. Do you think a non-proportional change in the linear dimensions of a two-dimensional figure will have the same effect on perimeter and area as proportional change? Explain your reasoning.

Answers will vary.

5. Consider the following rectangles with the dimensions shown.

Complete the table to determine how adding two or three inches to each rectangle’s base and height affects its perimeter and area. The information for Rectangle 1 has been done for you.

<table>
<thead>
<tr>
<th>Original Rectangle</th>
<th>Rectangle Formed by Adding 2 Inches to Dimensions</th>
<th>Rectangle Formed by Adding 3 Inches to Dimensions</th>
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<tr>
<td><strong>Rectangle 1</strong></td>
<td><strong>Linear Dimensions</strong></td>
<td><strong>Perimeter (in.)</strong></td>
</tr>
<tr>
<td></td>
<td><strong>b = 5 in., h = 4 in.</strong></td>
<td><strong>2(5 + 4) = 18</strong></td>
</tr>
<tr>
<td></td>
<td><strong>b = 7 in., h = 6 in.</strong></td>
<td><strong>2(7 + 6) = 26</strong></td>
</tr>
<tr>
<td></td>
<td><strong>b = 8 in., h = 7 in.</strong></td>
<td><strong>2(8 + 7) = 54</strong></td>
</tr>
<tr>
<td><strong>Rectangle 2</strong></td>
<td><strong>Linear Dimensions</strong></td>
<td><strong>Perimeter (in.)</strong></td>
</tr>
<tr>
<td></td>
<td><strong>b = 6 in., h = 2 in.</strong></td>
<td><strong>2(6 + 2) = 16</strong></td>
</tr>
<tr>
<td></td>
<td><strong>b = 8 in., h = 4 in.</strong></td>
<td><strong>2(8 + 4) = 24</strong></td>
</tr>
<tr>
<td></td>
<td><strong>b = 9 in., h = 5 in.</strong></td>
<td><strong>2(9 + 5) = 28</strong></td>
</tr>
<tr>
<td><strong>Rectangle 3</strong></td>
<td><strong>Linear Dimensions</strong></td>
<td><strong>Perimeter (in.)</strong></td>
</tr>
<tr>
<td></td>
<td><strong>b = 3 in., h = 3 in.</strong></td>
<td><strong>2(3 + 3) = 18</strong></td>
</tr>
<tr>
<td></td>
<td><strong>b = 5 in., h = 5 in.</strong></td>
<td><strong>2(5 + 5) = 20</strong></td>
</tr>
<tr>
<td></td>
<td><strong>b = 6 in., h = 6 in.</strong></td>
<td><strong>2(6 + 6) = 24</strong></td>
</tr>
</tbody>
</table>
6. Describe how a non-proportional change in the linear dimensions of a rectangle affects its perimeter.

a. What happens to the perimeter of a rectangle when 2 inches are added to its dimensions?
   When 2 inches are added to the dimensions of a rectangle, the resulting perimeter is 8 inches greater than the original perimeter.

b. What happens to the perimeter of a rectangle when 3 inches are added to its dimensions?
   When 3 inches are added to the dimensions of a rectangle, the resulting perimeter is 12 inches greater than the original perimeter.

c. What would happen to the perimeter of a rectangle if 4 inches are added to its dimensions?
   If 4 inches were added to the dimensions of a rectangle, then its perimeter would increase by 16 inches.

d. Describe how you think the perimeter of the resulting rectangle would compare to the perimeter of a 4 × 10 rectangle if the dimensions of the original rectangle were reduced by 2 inches. Then, determine the perimeter of the resulting rectangle.
   The perimeter of the resulting rectangle, 20 inches, would be 8 inches less than the perimeter of the original rectangle, 28 inches.
   Dimensions of Resulting Rectangle (inches):
   Base = 4 − 2 = 2
   Height = 10 − 2 = 8
   Perimeter of Resulting Rectangle (inches):
   2(2 + 8) = 20

e. Given that a rectangle’s original dimensions change by x units, generalize the change in the perimeter in terms of x?
   Given that a rectangle’s original dimensions change by x, its perimeter will change by a factor of 4x.
7. Describe how a non-proportional change in the linear dimensions of a rectangle affects its area.
   a. What happens to the area of a rectangle when its dimensions increase by 2 inches?
      Answers will vary.
      When the dimensions of a rectangle increase by 2 inches, the resulting area increases, but I do not see a clear cut pattern of increase as was the case with proportional change in linear dimensions.

   b. What happens to the area of a rectangle when its dimensions increase by a factor of 3?
      Answers will vary.
      When the dimensions of a rectangle increase by 3 inches, the resulting area increases, but I do not see a clear cut pattern of increase as was the case with proportional change in linear dimensions.

   c. What would happen to the area of a rectangle when its dimensions increase by a factor of 4?
      Answers will vary.
      When the dimensions of a rectangle increase by 4 inches, the resulting area will increase, but there will probably not be a clear cut pattern of increase as was the case with proportional change in linear dimensions.
d. Describe how you think the area of the resulting rectangle would compare to the area of a $4 \times 10$ rectangle if the dimensions of the original rectangle were reduced by 2 inches. Then, determine the area of the resulting rectangle.

Answers will vary.

The area of the resulting rectangle, 16 square inches, is less than the area of the original rectangle, 40 square inches. I do not see a clear cut pattern of decrease as was the case with proportional change in linear dimensions.

Dimensions of Resulting Rectangle (inches):
Base $= 4 - 2 = 2$
Height $= 10 - 2 = 8$

Area of Resulting Rectangle (square inches):
$2(8) = 16$

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e. What happens to the area when the dimensions of a rectangle change non-proportionally?

Answers will vary.

When the dimensions of a rectangle change non-proportionally, the resulting area will change, but there is not a clear cut pattern of increase or decrease as was the case with proportional change in linear dimensions.

f. Summarize the change in a rectangle's area caused by a non-proportional change in its linear dimensions and the change in a rectangle's area caused by a proportional change in its linear dimensions.

When the dimensions of a rectangle change non-proportionally, the resulting area will change, but there is not a clear cut pattern of increase or decrease as was the case with proportional change in linear dimensions.

When the dimensions of a rectangle change proportionally by a factor of $k$, its area changes by a factor of $k^2$. 
Problem 4
Students are given two sets of coordinates and use them to draw and label a quadrilateral on the coordinate plane that satisfies the constraint regarding the rule of thumb given in Problem 1 and at the same time, accommodates a 100-person occupancy. There is more than one correct solution to this problem situation.

Grouping
Have students complete Question 1 with a partner. Then have students share their responses as a class.

Problem 4 Party Time!
1. Use the coordinate plane shown to draw a floor plan for a space that would allow Marissa to invite 100 people including herself to her graduation party, given the rule of thumb for maximum occupancy. Two coordinates of the location are provided measured in feet: (25, 30) and (65, 75). Show your work.

Answers will vary.
One possible location for the other two coordinates are (23, 112) and (–17, 70). Using these coordinates, the space would be a square that has a side length of 60.21. The location of the other two points can be determined by using the given segment as a side of a square. The slope formula can be used to calculate the slope of the line segment, and that slope can be used to determine the slope of a line perpendicular to the given line segment. Once that slope is determined, it can be used to locate the two unknown points.

In order to meet the rule-of-thumb regulation for maximum occupancy of 100 people, the space for the party needs to have an area of 60.21 \times 60.21, or 3625 square feet. This square location will satisfy the constraint of 100 people.
60.21^2 = 3625 square feet
3625 \div 36 \approx 100.69

Be prepared to share your solutions and methods.
Is quadrilateral $ABCD$ a square? Show your work.

Yes, quadrilateral $ABCD$ is a square. All four sides are congruent and adjacent sides are perpendicular.

$AB = \sqrt{(2 - -3)^2 + (15 - 3)^2}$
$= \sqrt{5^2 + (12)^2}$
$= \sqrt{25 + 144} = \sqrt{169} = 13$

$BC = \sqrt{(-3 - 9)^2 + (3 - -2)^2}$
$= \sqrt{(-12)^2 + (5)^2}$
$= \sqrt{144 + 25} = \sqrt{169} = 13$

$CD = \sqrt{(14 - 9)^2 + (10 - -2)^2}$
$= \sqrt{(5)^2 + (12)^2}$
$= \sqrt{25 + 144} = \sqrt{169} = 13$

$AD = \sqrt{(14 - 2)^2 + (10 - 15)^2}$
$= \sqrt{(12)^2 + (-5)^2}$
$= \sqrt{144 + 25} = \sqrt{169} = 13$

Slope of $AB$: $m = \frac{15 - 3}{2 - -3} = \frac{12}{5}$

Slope of $BC$: $m = \frac{-2 - 3}{9 - -3} = -\frac{5}{12}$

Slope of $CD$: $m = \frac{-2 - 10}{9 - 14} = -\frac{12}{5}$

Slope of $AD$: $m = \frac{10 - 15}{14 - 2} = -\frac{5}{12}$
Looking at Something Familiar in a New Way

Area and Perimeter of Triangles on the Coordinate Plane

LEARNING GOALS

In this lesson, you will:

• Determine the perimeter of triangles on the coordinate plane.
• Determine the area of triangles on the coordinate plane.
• Determine and describe how proportional and non-proportional changes in the linear dimensions of a triangle affect its perimeter and area.
• Explore the effects that doubling the area has on the properties of a triangle.

ESSENTIAL IDEAS

• Rigid motion is used to change the position of triangles on the coordinate plane.
• Rigid motion is used to determine the area and perimeter of rectangles.
• The slope formula and the Distance Formula are used to determine the area and perimeter of triangles on the coordinate plane.
• Doubling the height or the length of the base doubles the area of the triangle.
• When the dimensions of a plane figure change proportionally by a factor of \( k \), its perimeter changes by a factor of \( k \), and its area changes by a factor of \( k^2 \).
• When the dimensions of a plane figure change non-proportionally, its perimeter and area increase or decrease non-proportionally.

TEXAS ESSENTIAL KNOWLEDGE AND SKILLS FOR MATHEMATICS

(3) Coordinate and transformational geometry. The student uses the process skills to generate and describe rigid transformations (translation, reflection, and rotation) and non-rigid transformations (dilations that preserve similarity and reductions and enlargements that do not preserve similarity). The student is expected to:

(B) determine the image or pre-image of a given two-dimensional figure under a composition of rigid transformations, a composition of non-rigid transformations, and a composition of both, including dilations where the center can be any point in the plane.

(10) Two-dimensional and three-dimensional figures. The student uses the process skills to recognize characteristics and dimensional changes of two- and three-dimensional figures. The student is expected to:

(B) determine and describe how changes in the linear dimensions of a shape affect its perimeter, area, surface area, or volume, including proportional and non-proportional dimensional change.
Overview

Rigid motion is used to explore the perimeter and area of triangles on the coordinate plane in this lesson. Students begin the lesson by explaining the connections between the area of a parallelogram formula and the area of a triangle formula. Then, given the coordinates of a triangle, they will compute the perimeter and area using necessary formulas. Students then analyze whether using different sides of the triangle as the base affect the area of the triangle. Finally, the idea of doubling the height or length of a triangle to double the area is also explored. Students also investigate how proportional and non-proportional changes to the linear dimensions of a plane figure affect its perimeter and area.
Warm Up

Four points and their coordinates are given.

1. Compute the perimeter of triangle $ABC$.
   
   $AB = \sqrt{(4 - 0)^2 + (0 - 6)^2}$
   
   $= \sqrt{4^2 + 6^2}$
   
   $= \sqrt{16 + 36} = \sqrt{52} \approx 7.2$
   
   $BC = \sqrt{(0 - 0)^2 + (8 - 6)^2}$
   
   $= \sqrt{0^2 + 14^2}$
   
   $= \sqrt{0 + 196} = \sqrt{196} = 14$
   
   $AC = \sqrt{(0 - 4)^2 + (8 - 0)^2}$
   
   $= \sqrt{(-4)^2 + 8^2}$
   
   $= \sqrt{16 + 64} = \sqrt{80} \approx 8.9$
   
   The approximate perimeter of triangle $ABC$ is $7.2 + 14 + 8.9 = 30.1$.
   
   The actual perimeter of triangle $ABC$ is $\sqrt{52} + 14 + \sqrt{80}$.

2. Compute the area of triangle $ABC$.
   
   $A = \frac{1}{2}bh$
   
   $A = \frac{1}{2}(14)(4) = 28$
   
   The area of triangle $ABC$ is 28 square units.

3. Was it easier to compute the perimeter or area of triangle $ABC$? Explain your reasoning.
   
   It was easier to compute the area because the length and the height of the triangle are whole numbers. To compute the actual perimeter, you have to add radicals.
Looking at Something Familiar in a New Way
Area and Perimeter of Triangles on the Coordinate Plane

LEARNING GOALS

In this lesson, you will:

- Determine the perimeter of triangles on the coordinate plane.
- Determine the area of triangles on the coordinate plane.
- Determine and describe how proportional and non-proportional changes in the linear dimensions of a triangle affect its perimeter and area.
- Explore the effects that doubling the area has on the properties of a triangle.

One of the most famous stretches of ocean in the Atlantic is an area that stretches between the United States, Puerto Rico, and Bermuda known as the Bermuda Triangle.

A heavily traveled area by planes and ships, it has become famous because of the many stories about ships and planes lost or destroyed as they moved through the Triangle.

For years, the Bermuda Triangle was suspected of having mysterious, supernatural powers that fatally affected all who traveled through it. Others believe natural phenomena, such as human error and dangerous weather, are to blame for the incidents.
Problem 1
Students derive the area of a triangle formula using the area of a rectangle. They are given three coordinates and will graph a triangle which lies in the first and second quadrants. Next, they compute the perimeter and area of the triangle using formulas. Then they transform the triangle, calculate the perimeter and area again, and conclude the results remained unaltered through the translation. Finally, students are given four different students’ worked examples of transformations to discover that there are many ways to transform pre-images but the perimeter and area are always preserved through the translations.

Grouping
• Complete Question 1 as a class.
• Have students complete Questions 2 and 3 with a partner. Then have students share their responses as a class.

Guiding Questions for Discuss Phase, Question 1
• How is the area of the parallelogram related to the area of the triangle?
• Is there any other way to determine the formula using the parallelogram?

PROBLEM Determining the Area of a Triangle

1. The formula for calculating the area of a triangle can be determined from the formula for the area of a parallelogram.

   a. Explain how the formula for the area of a triangle is derived using the given parallelogram.
   The area of parallelogram ABCD is equal to the base times the height. Diagonal AC divides the parallelogram into two congruent triangles, each with the same base and height as the parallelogram. Therefore, the area of each triangle is half the area of the parallelogram.

   b. Write the formula for the area of a triangle.
   \[ A = \frac{1}{2}bh \]

2. Graph triangle ABC with vertices A(−7.5, 2), B(−5.5, 13), and C(2.5, 2). Then, determine its perimeter.

   \[ AB = \sqrt{(-5.5 - (-7.5))^2 + (13 - 2)^2} = \sqrt{2^2 + 11^2} = \sqrt{4 + 121} = \sqrt{125} = 5\sqrt{5} \]

   \[ BC = \sqrt{(2.5 - (-5.5))^2 + (2 - 13)^2} = \sqrt{8^2 + (-11)^2} = \sqrt{64 + 121} = \sqrt{185} \]

   \[ AC = 10 \]

   Perimeter of triangle ABC = \( AB + BC + AC \)
   \[ = 5\sqrt{5} + \sqrt{185} + 10 \]
   \[ = 34.8 \]

   The perimeter of triangle ABC is approximately 34.8 units.
Guiding Questions for Share Phase, Questions 2 and 3

- How would you describe the orientation of this triangle?
- What is the length of the base of this triangle?
- What is the height of this triangle?
- What formula is used to determine the perimeter of triangle ABC?
- What is the difference between an actual perimeter and an approximate perimeter?
- Can you determine the actual perimeter or an approximate perimeter in this situation? Explain.
- What unit of measure is associated with the perimeter in this situation?
- What formula is used to determine the area of triangle ABC?
- Is the area you determined the actual area or an approximate area? Explain.
- What unit of measure is associated with the area in this situation?

3. Determine the area of triangle ABC.
   a. What information is needed about triangle ABC to determine its area?
      To determine the area of the triangle, I need to know the length of the base and the height.
   b. Arlo says that line segment AB can be used as the height. Trisha disagrees and says that line segment BC can be used as the height. Randy disagrees with both of them and says that none of the line segments that make up the triangle can be used as the height. Who is correct? Explain your reasoning.
      Randy is correct. The height of a triangle must be a line segment drawn from a vertex perpendicular to the opposite side. Line segments AB and BC cannot be used as the height because they are not perpendicular to any base.
   c. Draw a line segment that represents the height of triangle ABC. Label the line segment BD. Then, determine the height of triangle ABC.
      The y-coordinate of point B is 13. Point D has the same y-coordinate as points A and C, which is 2. So, the length of segment BD is 13 – 2, or 11 units.
   d. Determine the area of triangle ABC.
      \[ A = \frac{1}{2}bh \]
      \[ = \frac{1}{2}(10)(11) \]
      \[ = 55 \]
      The area of triangle ABC is 55 square units.
Grouping
Have students complete Questions 4 and 5 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Questions 4 and 5

- How is the orientation of this triangle different than the orientation of the triangle in Question 2?
- How will you determine the length of the base and the height of this triangle?
- Did you use the origin in your transformation? If so, how?
- Did you use any axes in your transformation? If so, how?
- Were the calculations easier after the transformation? Explain.
- Did your transformation preserve the perimeter and area of the triangle?
- Did your classmates use the same transformation?
- Did your classmates’ transformations also preserve the perimeter and area of the triangle?

4. Perform one or more transformations to help you determine the perimeter and the area more efficiently.

a. Transform triangle ABC on the coordinate plane. Label the image A’B’C’.
Describe the transformation(s) completed and explain your reasoning.

Answers will vary.
I vertically translated the triangle down 2 units so that the base, line segment AC, is on the x-axis. I, then, horizontally translated the triangle to the right 5.5 units so that point B is on the y-axis.

b. Determine the perimeter of triangle A’B’C’.
\[ A'B' = \sqrt{(0 - (-2))^2 + (11 - 0)^2} \]
\[ = \sqrt{4 + 121} \]
\[ = \sqrt{125} = 5\sqrt{5} \]
\[ B'C' = \sqrt{(8 - 0)^2 + (0 - 11)^2} \]
\[ = \sqrt{64 + 121} \]
\[ = \sqrt{185} \]

Points A’ and C’ have the same y-coordinate, so I can subtract the x-coordinates to calculate the length of line segment A’C’.
\[ A'C' = 8 - (-2) \]
\[ = 10 \]

Perimeter of triangle A’B’C’ = A’B’ + B’C’ + A’C’
\[ = 5\sqrt{5} + \sqrt{185} + 10 \]
\[ \approx 34.8 \]

The perimeter of triangle A’B’C’ is approximately 34.8 units.
c. Determine the area of triangle $A'B'C'$. Be sure to label the height on the coordinate plane as line segment $B'D'$.

The $y$-coordinate of point $B'$ is 11. Point $D'$ has the same $y$-coordinate as points $A'$ and $C'$, which is 0. So, the length of segment $B'D'$ is $11 - 0$, or 11 units.

$$A = \frac{1}{2}bh$$

$$= \frac{1}{2}(10)(11)$$

$$= 55$$

The area of triangle $A'B'C'$ is 55 square units.

5. Compare the perimeters and the areas of triangles $ABC$ and $A'B'C'$.

a. What do you notice about the perimeters and the areas of both triangles?

Both triangles have the same perimeter and area.

b. Use what you know about transformations to explain why this occurs.

Translating a figure results in a figure that is congruent to the original figure. Two figures that are congruent have the same area and perimeter.
Grouping
Have students complete Question 6 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Question 6
• How did you determine the distance each triangle was translated?
• Which student’s transformation do you prefer? Explain.
• Which transformation is least desirable? Explain.
• Is the area preserved in each transformation?
• Is the perimeter preserved in each transformation?

6. Mr. Young gives his class triangle DEF and asks them to determine the area and perimeter.

Four of his students decide to first transform the figure and then determine the perimeter and the area. Their transformations are shown.
a. Describe the transformation(s) each student made to triangle DEF.

- Michael vertically translated triangle DEF up 9 units.
- Angelica vertically translated triangle DEF up 9 units, then horizontally translated it to the left 3 units.
- Juan vertically translated triangle DEF up 9 units, then horizontally translated it to the left 3 units. Then, he reflected it over the y-axis.
- Isabel horizontally translated triangle DEF to the left 3 units.

b. Whose method is most efficient? Explain your reasoning.

Answers will vary. Juan’s method is most efficient because it transforms the triangle into Quadrant I so that all his coordinates are positive and two line segments of the triangle are on the axes.

c. What do you know about the perimeters and areas of all the students’ triangles? Explain your reasoning.

The perimeters and the areas of all the triangles are the same. I know this because each student used translations and reflections and I know that translations and reflections result in congruent figures.
Problem 2
Any of the three sides of a triangle can be considered the base of the triangle. A triangle’s orientation is along a diagonal on a graph. Students calculate the area of the triangle three times, each time using a different base to conclude the area remains unaltered.

Grouping
Have students complete Questions 1 through 5 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Questions 1 through 5
- If line segment AC is thought of as the base of the triangle, how would you describe the location of the height?
- How is the location of the height determined?
- What is the point-slope equation for a line?
- How are the coordinates of the endpoints of the line segment representing the height determined?
- What are the equations for the two lines intersecting at the endpoint of the line segment representing the height?
- Is the area an approximation? Why or why not?

PROBLEM  Which Way Is Up?

1. Graph triangle ABC with vertices A(2, 5), B(10, 9), and C(6, 1). Determine the perimeter.

Perimeter of triangle ABC = AB + BC + AC

\begin{align*}
AB &= \sqrt{(10 - 2)^2 + (9 - 5)^2} \\
&= \sqrt{8^2 + 4^2} \\
&= \sqrt{64 + 16} = \sqrt{80} = 4\sqrt{5} \\
BC &= \sqrt{(10 - 6)^2 + (9 - 1)^2} \\
&= \sqrt{4^2 + 8^2} \\
&= \sqrt{16 + 64} = \sqrt{80} = 4\sqrt{5} \\
AC &= \sqrt{(2 - 6)^2 + (5 - 1)^2} \\
&= \sqrt{(-4)^2 + 4^2} \\
&= \sqrt{16 + 16} = \sqrt{32} = 4\sqrt{2} \\
\end{align*}

The perimeter of triangle ABC is approximately 23.5 units.
2. To determine the area, you will need to determine the height. How will determining the height of this triangle be different from determining the height of the triangle in Problem 1?

In Problem 1, the base was a horizontal line segment and the height was a vertical line segment. So, point D had the same y-coordinate as points A and C. The height was calculated as the difference of the y-coordinates.

In this triangle, the base and the height are neither horizontal nor vertical. So, the calculations will be different.

To determine the height of this triangle, you must first determine the endpoints of the height. Remember that the height must always be perpendicular to the base.

Let’s use \( \overline{AC} \) as the base of triangle ABC. Determine the coordinates of the endpoints of height \( \overline{BD} \).

- **Calculate the slope of the base.**
  \[ m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 5}{6 - 2} = -\frac{4}{4} = -1 \]

- **Determine the slope of the height.**
  \[ m = 1 \]

- **Determine the equation of the base.**
  Base \( \overline{AC} \) has a slope of \(-1\) and passed through point A (5, 2).
  \[ (y - y_1) = m(x - x_1) \]
  \[ (y - 2) = -1(x - 5) \]
  \[ y = -x + 7 \]

- **Determine the equation of the height.**
  Height \( \overline{BD} \) has a slope of 1 and passed through point B (10, 9).
  \[ (y - y_2) = m(x - x_2) \]
  \[ (y - 9) = 1(x - 10) \]
  \[ y = x - 1 \]

- **Solve the system of equations to determine the coordinates of the point of intersection.**
  \[ \begin{align*}
  x - 1 &= -x + 7 \\
  y &= x - 1
  \end{align*} \]
  \[ \begin{align*}
  2x &= 8 \\
  y &= 4 - 1
  \end{align*} \]
  \[ \begin{align*}
  x &= 4 \\
  y &= 3
  \end{align*} \]

The coordinates of point D are (4, 3).
3. Graph the point of intersection on the coordinate plane and label it point D. Draw line segment $BD$ to represent the height. See coordinate plane.

4. Determine the area of triangle $ABC$.
   a. Determine the length of height $BD$.
      
      $$BD = \sqrt{(10 - 4)^2 + (9 - 3)^2}$$
      
      $$= \sqrt{(6)^2 + (6)^2}$$
      
      $$= \sqrt{36 + 36}$$
      
      $$= \sqrt{72}$$
      
      $$= 6\sqrt{2}$$
      
   b. Determine the area of triangle $ABC$.
      
      $$A = \frac{1}{2}bh$$
      
      $$= \frac{1}{2}(4\sqrt{2})(6\sqrt{2})$$
      
      $$= 24$$
      
      The area of triangle $ABC$ is 24 square units.

5. You know that any side of a triangle can be the base of the triangle. Predict whether using a different side as the base will result in a different area of the triangle. Explain your reasoning.
   
   Answers will vary.
   
   No. Using a different side as the base will not result in a different area. The lengths of the base and height will be different but their product will be the same.
   
   Let’s consider your prediction.

6. Triangle $ABC$ is given on the coordinate plane. This time, let’s consider side $AB$ as the base.

   - What is the point-slope equation for a line?
   - How are the coordinates of the endpoints of the line segment representing the height determined?
   - What are the equations for the two lines intersecting at the endpoint of the line segment representing the height?
   - Is the area an approximation? Why or why not?
a. Let point $D$ represent the intersection point of the height, $CD$, and the base. Determine the coordinates of point $D$.

Slope of base $\overline{AB}$:

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{9 - 5}{10 - 2} = \frac{4 - 1}{8 - 2}$$

Slope of height $\overline{CD}$:

Equation of height $\overline{CD}$:

$$(y - y_1) = m(x - x_1)$$

$(y - 1) = -2(x - 6)$

$$y = -2x + 13$$

Equation of base $\overline{AB}$:

$$(y - y_1) = m(x - x_1)$$

$(y - 5) = \frac{3}{2}(x - 2)$

$$y = \frac{3}{2}x + 4$$

Solution of the system of equations:

$-2x + 13 = \frac{1}{2}x + 4$

$9 = \frac{5}{2}x$

$\frac{18}{5} = x$

$y = -2(\frac{18}{5}) + 13$

$y = \frac{29}{5}$

The coordinates of point $D$ are $\left(\frac{18}{5}, \frac{29}{5}\right)$.

b. Determine the height of triangle $ABC$.

$$CD = \sqrt{\left(6 - \frac{18}{5}\right)^2 + \left(1 - \frac{29}{5}\right)^2}$$

$$= \sqrt{\left(\frac{12}{5}\right)^2 + \left(\frac{-24}{5}\right)^2}$$

$$= \sqrt{\frac{144}{25} + \frac{576}{25}} = \sqrt{\frac{720}{25}} = \frac{\sqrt{720}}{5}$$

$$= 2\sqrt{5}$$

$$\frac{\sqrt{25}}{5}$$

$$= 2\sqrt{5}$$

$$= 24$$

The area of triangle $ABC$ is 24 square units.

c. Determine the area of triangle $ABC$.

$$A = \frac{1}{2}bh$$

$$= \frac{1}{2}(4/5) \frac{\sqrt{720}}{5}$$

$$= 24$$

The area of triangle $ABC$ is 24 square units.
7. Triangle **ABC** is graphed on the coordinate plane. Determine the area of triangle **ABC** using **BC** as the base.

Slope of base \( BC \) = \( \frac{y_2 - y_1}{x_2 - x_1} \)  
\[ = \frac{9 - 1}{10 - 6} = \frac{8}{4} = 2 \]

Slope of height \( AD \) = \( -\frac{1}{2} \)

Equation of base \( BC \):  
\[ (y - y_1) = m(x - x_1) \]
\[ (y - 1) = 2(x - 6) \]
\[ y = 2x - 11 \]

Equation of height \( AD \):  
\[ (y - y_1) = m(x - x_1) \]
\[ (y - 5) = -\frac{1}{2}(x - 2) \]
\[ y = -\frac{1}{2}x + 6 \]

Solution of the system of equations:
\[ 2x - 11 = -\frac{1}{2}x + 6 \]
\[ 5x = 17 \]
\[ x = \frac{34}{5} \]

\[ y = 2\left(\frac{34}{5}\right) - 11 \]
\[ y = \frac{13}{5} \]

The coordinates of point **D** are \( \left(\frac{34}{5}, \frac{13}{5}\right) \)

Height of triangle **ABC**:  
\[ AD = \sqrt{\left(\frac{34}{5} - 2\right)^2 + \left(\frac{13}{5} - 5\right)^2} \]
\[ = \sqrt{\left(\frac{24}{5}\right)^2 + \left(-\frac{12}{5}\right)^2} \]
\[ = \sqrt{\frac{576}{25} + \frac{144}{25}} \]
\[ = \sqrt{\frac{720}{25}} \]
\[ = \frac{\sqrt{720}}{5} \]

Area of triangle **ABC**:  
\[ A = \frac{1}{2}bh \]
\[ = \frac{1}{2}(4\sqrt{6}) \frac{\sqrt{13}}{5} \]
\[ = 24 \]

The area of triangle **ABC** is 24 square units.
Problem 3
A scenario is used to show that when a triangle is translated, area and perimeter are preserved. Students also calculate the cost of a project given the expense per linear foot of fencing. Again, transformations are emphasized to make calculations more efficient.

Grouping
Have students complete Questions 1 through 5 with a partner. Then have students share their responses as a class.

8. Compare the three areas you determined for triangle ABC. Was your prediction in Question 5 correct?
   Yes. My prediction was correct. The areas are the same no matter which side of the triangle I used as a base.

Problem 3
It's a Dog's Life
Joseph plans to fence in a corner of his property so his dog can exercise there. Consider the triangular space shown. Each of the three corners of the space is labeled with coordinates and helps define the dimensions, in feet, of the fenced portion of land.
Guiding Questions for Share Phase, Questions 1 through 5

- Is area or perimeter more helpful when determining the cost of the project? Why?
- Is the perimeter exact or approximate? Why?
- What unit of measure is used to describe the perimeter?
- Is area or perimeter more helpful when determining the amount of space the dog will have to exercise? Why?
- Is the area exact or approximate? Why?
- What unit of measure is used to describe the area?
- How many different translations are possible to locate one vertex at the origin?

1. Fencing costs $15 per linear foot. How much will this project cost Joseph? Show your work.

   \[ AB = \sqrt{(-2 - (-3))^2 + (2 - (-3))^2} \]
   \[ = \sqrt{(5)^2 + (5)^2} \]
   \[ = \sqrt{25 + 25} \]
   \[ = \sqrt{50} = 7.1 \]

   \[ BC = \sqrt{(-3 - 5)^2 + (-3 - (-5))^2} \]
   \[ = \sqrt{(-8)^2 + (2)^2} \]
   \[ = \sqrt{64 + 4} \]
   \[ = \sqrt{68} = 8.2 \]

   \[ AC = \sqrt{(2 - 5)^2 + (2 - (-5))^2} \]
   \[ = \sqrt{(-3)^2 + (7)^2} \]
   \[ = \sqrt{9 + 49} \]
   \[ = \sqrt{58} \approx 7.6 \]

Perimeter of triangle \( ABC = 7.1 + 8.2 + 7.6 = 22.9 \) feet

This project will cost Joseph approximately \( 22.9 \times 15 \), or $343.50.
2. Calculate the amount of space Joseph’s dog will have to exercise. Show your work.

Slope of base $AB = \frac{-3 - 2}{-3 - 2} = \frac{-5}{-5} = 1$

Slope of height $AD = -1$

Equation of base $AB$: $y - 2 = 1(x - 2)$

$y - 2 = x - 2$

$y = x$

Equation of height $CD$: $y - 5 = -1(x - 0)$

$y + 5 = -x + 5$

$y = -x$

Solution of the system of equations:

$x = -x$

$2x = 0$

$x = 0$

The coordinates of point $D$ are $(0, 0)$.

Height of triangle $ABC$:

$AC = \sqrt{(0 - 5)^2 + (0 - (-5))^2}$

$= \sqrt{(-5)^2 + (5)^2}$

$= \sqrt{25 + 25}$

$= \sqrt{50} \approx 7.1$

Area of triangle $ABC = \frac{1}{2} \times 7.1 \times 7.1 = 25.2$ square feet

Joseph’s dog will have approximately 25.2 square feet to exercise.
3. Compare your answer to Question 2 with your classmates’ answers. Was your solution path the same or different from your classmates’ solution paths?
   Answers will vary.
   Student responses may include:
   Some classmates vertically and horizontally translated the triangle before they calculated the area.

4. Describe how transformations could be used to make the calculations more efficient.
   Answers will vary.
   Student responses may include:
   The vertices of the triangle could be translated vertically and horizontally such that one of the vertices is on the origin (0, 0) or a side of the triangle could lie on one of the axes.

5. If the same vertical and horizontal translations were performed on the three vertices of the triangle, describe how this would affect the perimeter and the area of the triangle.
   When translations are performed on a triangle, the perimeter and the area are preserved.
Problem 4
Students apply what they have learned to map the Bermuda Triangle to the coordinate plane given the Triangle's perimeter, area, and two of the side distances.

Grouping
Have students complete the problem with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Problem 4
- What city did you locate first? Why?
- How did you decide where to place the first city?
- What city did you locate second? Why?
- How did you decide where to place the second city?
- How did you decide where to place the third city?
- How is the distance from Puerto Rico to Bermuda determined?
- What is the distance from Puerto Rico to Bermuda?
- How is the area formula helpful?
- How is the Pythagorean Theorem helpful?

The Bermuda Triangle is an imaginary triangle connecting Miami, Florida, to San Juan, Puerto Rico, to Bermuda. It has a rich history of suspected paranormal activities, which include the disappearances of boats and aircraft.

Consider these approximate measurements:
- The distance from Miami to San Juan is 1060 miles.
- The distance from Miami to Bermuda is 1037 miles.
- The perimeter of the Bermuda Triangle is 3078 miles.
- The Bermuda Triangle is a region of 454,000 square miles.
Place the Bermuda Triangle on the coordinate plane and provide the coordinates for each of the three vertices. Show your work.

Let point \( P \) represent Puerto Rico, point \( B \) represent Bermuda, and point \( M \) represent Miami.

I can orient the axes in any position, so I chose to place Puerto Rico at the origin. Point \( P \) has coordinates \((0, 0)\). I then rotated the map so that Bermuda is directly above Puerto Rico. The \( x \)-coordinate of point \( B \) is 0. To determine the \( y \)-coordinate, I need to calculate the distance between points \( B \) and \( P \).

Calculate the distance between Bermuda and Puerto Rico.

\[
\text{Perimeter} = MB + MP + BP \\
3078 = 1060 + 1037 + BP \\
BP = 981
\]

So, the \( y \)-coordinate of point \( B \) is 981. Point \( B \) is \((0, 981)\).

Consider \( BP \) as the base. I can calculate the corresponding height.

\[
A = \frac{1}{2}bh \\
454,000 = \frac{1}{2}(981)h \\
h = 925.59
\]

So, I know the \( x \)-coordinate of point \( M \) is \(-925.59\).

To calculate the \( y \)-coordinate of point \( M \), use the Pythagorean Theorem. A smaller triangle with the height as one leg and \( MP \) as the hypotenuse.

\[
925.59^2 + y^2 = 1037^2 \\
856,716.8481 + y^2 = 1,075,369 \\
y^2 = 218,652.1519 \\
y = 467.60
\]

So, the \( y \)-coordinate of point \( M \) is 467.60.
Problem 5
Students investigate again the effects of proportional and non-proportional changes on perimeter and area—this time with triangles. A triangle is drawn on the coordinate plane and when the height is doubled, the area is also doubled. Students review the solution to determine an error that was made. The translations and calculations are both correct, but the coordinates of point $A'$ are incorrect.

Grouping
Have students complete Questions 1 through 4 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Questions 1 through 4
- Do you think that proportional and non-proportional changes will have similar effects on triangles as they did on rectangles? Explain.
- Did Emilio identify the correct coordinates for each of the new vertices?

PROBLEM 5 Reaching for New Heights, and Bases . . . Again
Recall that you have investigated how proportional and non-proportional changes in the linear dimensions of a rectangle affect its perimeter and area.

1. Describe how adding 4 units to the side lengths of a triangle will affect the perimeter of the resulting triangle. Provide an example, determine its perimeter, and explain your reasoning.
   Adding 4 units to the side lengths of a triangle will result in a triangle with a perimeter that is $3 \times 4$, or 12 units greater.
   For example, adding 4 units to the side lengths of a triangle with side lengths of 6, 7, and 11 inches results in a triangle with side lengths of 10, 11, and 15 inches. The perimeter of the resulting triangle, 36 inches, is $3 \times 4$, or 12 inches greater than the perimeter of the original triangle, 24 inches.
   Perimeter of Original Triangle (inches) $6 + 7 + 11 = 24$
   Perimeter of Resulting Triangle (inches): $10 + 11 + 15 = 36$
   Difference between Resulting Perimeter and Original Perimeter (inches): $36 - 24 = 12$

Recall that increasing the linear dimensions of a rectangle by a factor of $k$ increases its area by a factor of $k^2$. Does this relationship exist with non-right triangles?

2. How can you use constructions to determine, describe, and verify that increasing the side lengths of a non-right triangle by a factor of $k$ will increase its area by a factor of $k^2$? Explain your reasoning.
   I could begin by constructing a non-right triangle and a second non-right triangle by doubling each side length. Then, I could verify that the factor of increase for the base and height of the triangle is also 2. This step is necessary because base and height are used to calculate the area of a triangle. I know that doubling the base and height of the triangle increases its area by a factor of $2 \times 2$, or 4. If follows that doubling the side lengths of a non-right triangle also increases its area by a factor of $2 \times 2$, or 4.
3. Describe how multiplying the base and height of a triangle by a factor of 5 will affect
the area of the resulting triangle. Provide an example, determine its area, and explain
your reasoning.

Multiplying the base and height of a triangle by a factor of 5 will result in a triangle
that has an area $5 \times 5$, or 25 square units greater.

For example, consider a triangle with base 2 inches and height 3 inches. Multiplying
the base and height by 5 results in a base of 10 and a height of 15. The area of the
resulting triangle, 75 square inches, is 25 times greater than the area of the original
triangle, 3 square inches.

Area of Original Triangle (square inches):
\[
\frac{1}{2} (2)(3) = 3
\]

Area of Resulting Triangle (square inches):
\[
\frac{1}{2} (10)(15) = 75
\]

Ratio between Resulting Area and Original Area (square inches):
\[
\frac{75}{3} = 25
\]
4. Emilio’s class is given triangle ABC. Their teacher asks them to double the area of this triangle by manipulating the height. They must identify the coordinates of the new point, A’, and then determine the area. Emilio decides to first translate the triangle so it sits on grid lines to make his calculations more efficient. His work is shown.

Emilio is shocked to learn that he got this answer wrong. Explain to Emilio what he did wrong. Determine the correct answer for this question.

Although Emilio did do the transformations and the calculations correctly, he did not identify the correct coordinates for the new point. Although he is correct that his point A’ is at (−3, 6), that is not the correct point to use. He must translate this new point back to triangle ABC.

The correct coordinates of point A’ are (−5.5, 0.5).

Be prepared to share your solutions and methods.
Check for Students’ Understanding

Four points and their coordinates are given.

1. Compute the area of triangle ABC.
   \[ A = \frac{1}{2}bh \]
   \[ A = \frac{1}{2}(11)(6) = 33 \]
   The area of triangle ABC is 33 square units.

2. Is triangle ABC a right triangle? Show your work.
   No. Triangle ABC is not a right triangle.
   \[ AB = \sqrt{(0 - -2)^2 + (8 - -6)^2} \]
   \[ = \sqrt{2^2 + 14^2} \]
   \[ = \sqrt{4 + 196} = \sqrt{200} \]
   \[ BC = \sqrt{(-2 - 9)^2 + (-6 - -6)^2} \]
   \[ = \sqrt{(-11)^2 + 0^2} \]
   \[ = \sqrt{121 + 0} = \sqrt{121} = 11 \]
   \[ AC = \sqrt{(0 - 9)^2 + (8 - -6)^2} \]
   \[ = \sqrt{(-9)^2 + 14^2} \]
   \[ = \sqrt{81 + 196} = \sqrt{277} \]
   \[ a^2 + b^2 = c^2 \]
   \[ (\sqrt{121})^2 + (\sqrt{200})^2 = (\sqrt{277})^2 \]
   \[ 121 + 200 \neq 277 \]
Grasshoppers Everywhere!

Area and Perimeter of Parallelograms on the Coordinate Plane

LEARNING GOALS

In this lesson, you will:
- Determine the perimeter of parallelograms on a coordinate plane.
- Determine the area of parallelograms on a coordinate plane.
- Determine and describe how proportional and non-proportional changes in the linear dimensions of a parallelogram affect its perimeter and area.
- Explore the effects that doubling the area has on the properties of a parallelogram.

ESSENTIAL IDEAS
- Rigid motion is used to change the position of parallelograms on the coordinate plane.
- Rigid motion is used to determine the area and perimeter of parallelograms.
- The slope formula and the Distance Formula are used to determine the area and perimeter of parallelograms on the coordinate plane.
- The rectangle method is used to determine the area of parallelograms on the coordinate plane.
- Doubling the height or the length of the base doubles the area of the parallelogram.
- When the dimensions of a plane figure change proportionally by a factor of \( k \), its perimeter changes by a factor of \( k \), and its area changes by a factor of \( k^2 \).
- When the dimensions of a plane figure change non-proportionally, its perimeter and area increase or decrease non-proportionally.

TEXAS ESSENTIAL KNOWLEDGE AND SKILLS FOR MATHEMATICS

(3) Coordinate and transformational geometry. The student uses the process skills to generate and describe rigid transformations (translation, reflection, and rotation) and non-rigid transformations (dilations that preserve similarity and reductions and enlargements that do not preserve similarity). The student is expected to:

(B) determine the image or pre-image of a given two-dimensional figure under a composition of rigid transformations, a composition of non-rigid transformations, and a composition of both, including dilations where the center can be any point in the plane.
Overview
Rigid motion is used to explore the perimeter and area of parallelograms on the coordinate plane in this lesson. Students begin the lesson by explaining the connections between the area of a rectangle formula and the area of a parallelogram formula. Then, given a parallelogram, they will compute the perimeter and area using necessary formulas. Students then analyze whether using different sides of the parallelogram as the base affect the area of the parallelogram. Finally, the idea of doubling the height or length of a parallelogram to double the area is also explored. Students also investigate how proportional and non-proportional changes to the linear dimensions of a plane figure affect its perimeter and area.
Warm Up

Four points and their coordinates are given.

1. What is the length of the base of parallelogram $ABCD$?
   The length of the base of parallelogram $ABCD$ is 15 units.

2. What is the height of parallelogram $ABCD$?
   The height of parallelogram $ABCD$ is 11 units.

3. Compute the area of parallelogram $ABCD$.
   $A = bh$
   $= (15)(11) = 165$
   The area of parallelogram $ABCD$ is 165 square units.

4. What formula is needed to calculate the perimeter of parallelogram $ABCD$?
   The Distance Formula or the Pythagorean Theorem can be used to calculate the perimeter of parallelogram $ABCD$. 
Grasshoppers Everywhere!

Area and Perimeter of Parallelograms on the Coordinate Plane

LEARNING GOALS

In this lesson, you will:

- Determine the perimeter of parallelograms on a coordinate plane.
- Determine the area of parallelograms on a coordinate plane.
- Determine and describe how proportional and non-proportional changes in the linear dimensions of a parallelogram affect its perimeter and area.
- Explore the effects that doubling the area has on the properties of a parallelogram.

You wouldn’t think that grasshoppers could be dangerous. But they can damage farmers’ crops and destroy vegetation. In 2003, a huge number of grasshoppers invaded the country of Sudan, affecting nearly 1700 people with breathing problems.

Grasshopper invasions have been recorded in North America, Europe, the Middle East, Africa, Asia, and Australia. One of the largest swarms of grasshoppers—known as a “cloud” of grasshoppers—covered almost 200,000 square miles!
Problem 1
Students relate the area of a parallelogram to the area of a rectangle. They compute the perimeter and area of a parallelogram which lies in Quadrants III and IV using appropriate formulas.

Grouping
• Ask a student to read the information aloud and complete Question 1 as a class.
• Have students complete Question 2 with a partner. Then have students share their responses as a class.

Guiding Questions for Discuss Phase, Question 1
How is the area of a rectangle related to the area of a parallelogram?

PROBLEM Rectangle or . . . ?
You know the formula for the area of a parallelogram. The formula, \( A = bh \), where \( A \) represents the area, \( b \) represents the length of the base, and \( h \) represents the height, is the same formula that is used when determining the area of a rectangle. But how can that be if they are different shapes?

1. Use the given parallelogram to explain how the formula for the area of a parallelogram and the area of a rectangle can be the same.

In order to show the relationship between the parallelogram and a rectangle, I must manipulate the parallelogram and make it into a rectangle. First, I draw a straight line down from point \( A \), making triangle \( ABF \). I then move this triangle to the right side of the parallelogram, making triangle \( CDE \). This translation changes the parallelogram into a rectangle. Because the height, \( AF \), and the base, \( AD \) or \( FE \), remain the same, the area of the parallelogram is equal to the area of the rectangle.

2. Analyze parallelogram \( ABCD \) on the coordinate plane.

Could I transform this parallelogram to make these calculations easier?
Guiding Questions for Share Phase, Question 2

- How would you describe the orientation of this parallelogram?
- What is the length of the base of this parallelogram?
- What is the height of this parallelogram?
- What formula is used to determine the perimeter of parallelogram ABCD?
- Is a formula needed to determine the length of side BC? Why or why not?
- Is a formula needed to determine the length of side AD? Why or why not?
- Can you determine the actual perimeter or an approximate perimeter in this situation? Explain.
- What unit of measure is associated with the perimeter in this situation?
- Is the area you determined the actual area or an approximate area? Explain.
- What unit of measure is associated with the area in this situation?

a. Determine the perimeter of parallelogram ABCD.
\[ AB = \sqrt{(-2.5 - (-4.25))^2 + (-1.75 - (-3.5))^2} \]
\[ = \sqrt{(1.75)^2 + (1.75)^2} \]
\[ = \sqrt{3.0625 + 3.0625} \]
\[ = \sqrt{6.125} \]

Because line segment BC is horizontal, I can determine the length by subtracting the x-coordinates.
\[ BC = 2.25 - (-2.5) \]
\[ = 4.75 \]
\[ CD = \sqrt{(0.5 - 2.25)^2 + (-3.5 - (-1.75))^2} \]
\[ = \sqrt{(-1.75)^2 + (-1.75)^2} \]
\[ = \sqrt{3.0625 + 3.0625} \]
\[ = \sqrt{6.125} \]

Because line segment AD is horizontal, I can determine the length by subtracting the x-coordinates.
\[ AD = 0.5 - (-4.25) \]
\[ = 4.75 \]

Perimeter of parallelogram ABCD = \[ AB + BC + CD + AD \]
\[ = \sqrt{6.125} + 4.75 + \sqrt{6.125} + 4.75 \]
\[ \approx 14.45 \]

The perimeter of parallelogram ABCD is approximately 14.45 units.

b. To determine the area of parallelogram ABCD, you must first determine the height. Describe how to determine the height of parallelogram ABCD.
To determine the height of parallelogram ABCD, I must calculate the length of a perpendicular line segment from the base to a vertex opposite the base.

c. Ms. Finch asks her class to identify the height of parallelogram ABCD. Peter draws a perpendicular line from point B to AD, saying that the height is represented by BE. Tonya disagrees. She draws a perpendicular line from point D to BC, saying that the height is represented by DF. Who is correct? Explain your reasoning.
Both Peter and Tonya are correct. Either side BC or side AD can be used as the base. As long as their lines are perpendicular to the base they used, the heights should be the same.
Chapter 3  Perimeter and Area of Geometric Figures on the Coordinate Plane

Problem 2

Any of the four sides of a parallelogram can be considered the base of the parallelogram. A parallelogram's orientation is along a diagonal on a graph. Students calculate the area of the parallelogram twice, each time using a different base to conclude the perimeter and area remains unaltered.

Grouping

Have students complete Questions 1 through 3 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Questions 1 through 3

- If line segment CD is thought of as the base of the parallelogram, how would you describe the location of the height?
- How is the location of the height determined?
- What is the point-slope equation for a line?
- How are the coordinates of the endpoints of the line segment representing the height determined?
- What are the equations for the two lines intersecting at the endpoint of the line segment representing the height?
- Is the height an approximation or an exact answer? Why not?
- Is the area an approximation or an exact answer? Why?

Perimeter of parallelogram ABCD = AB + BC + CD + AD.

1. Graph parallelogram ABCD with vertices A (1, 1), B (7, -7), C (8, 0), and D (2, 8). Determine the perimeter.

<table>
<thead>
<tr>
<th>AB</th>
<th>CD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sqrt{(7-1)^2 + (-7-1)^2} )</td>
<td>( \sqrt{(2-8)^2 + (8-0)^2} )</td>
</tr>
<tr>
<td>( \sqrt{6^2 + (-8)^2} )</td>
<td>( \sqrt{(-6)^2 + (8)^2} )</td>
</tr>
<tr>
<td>( \sqrt{36 + 64} = \sqrt{100} = 10 )</td>
<td>( \sqrt{36 + 64} = \sqrt{100} = 10 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>BC</th>
<th>AD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sqrt{(8-7)^2 + (0-(-7))^2} )</td>
<td>( \sqrt{(2-1)^2 + (8-1)^2} )</td>
</tr>
<tr>
<td>( \sqrt{1^2 + (7)^2} )</td>
<td>( \sqrt{(1)^2 + (7)^2} )</td>
</tr>
<tr>
<td>( \sqrt{1 + 49} = \sqrt{50} )</td>
<td>( \sqrt{1 + 49} = \sqrt{50} )</td>
</tr>
</tbody>
</table>

The perimeter is approximately 34.14 units.

d. Determine the height of parallelogram ABCD.

Because the height is vertical, I can determine the length by subtracting the y-coordinates.

\[ BE = -1.75 - (-3.5) \quad \text{or} \quad DF = -1.75 - (-3.5) \]

\[ = 1.75 \]

e. Determine the area of parallelogram ABCD.

\[ A = bh \]

\[ = (4.75)(1.75) \]

\[ = 8.3125 \]

The area of parallelogram ABCD is 8.3125 square units.
2. Determine the area of parallelogram ABCD.
   a. Using \( \overline{CD} \) as the base, how will determining the height of this parallelogram be different from determining the height of the parallelogram in Problem 1?
   
   Because the base of this parallelogram is not horizontal, I cannot just draw a vertical height. I need to calculate the length of a perpendicular line segment that connects the base to a vertex opposite the base.

   b. Using \( \overline{CD} \) as the base, explain how you will locate the coordinates of point \( E \), the point where the base and height intersect.

   First, I will calculate the slope of the base. Then, I will determine the slope of the height. Because the base and the height must be perpendicular, this slope will be the negative reciprocal of the slope of the base. Next, I will use the slopes to determine the equation for the base and for a line that passes through point \( A \) and is perpendicular to the base. Finally, I will solve the system created by these two equations to determine the point of intersection. This point of intersection will provide the coordinates for point \( E \).

   c. Determine the coordinates of point \( E \). Label point \( E \) on the coordinate plane.

   **Slope of base \( \overline{CD} \):**
   
   \[
   m = \frac{y_2 - y_1}{x_2 - x_1}
   \]

   \[
   m = \frac{8 - 0}{2 - 8} = \frac{8}{-6} = -\frac{4}{3}
   \]

   **Slope of height \( \overline{AE} \):**

   \[
   m = \frac{3}{4}
   \]

   **Equation of base \( \overline{CD} \):**

   \[
   (y - y_1) = m(x - x_1)
   \]

   \[
   (y - 0) = -\frac{4}{3}(x - 2)
   \]

   **Equation of height \( \overline{AE} \):**

   \[
   (y - y_1) = m(x - x_1)
   \]

   \[
   (y - 1) = \frac{3}{4}(x - 1)
   \]

   Solution of the system of equations:

   \[
   \begin{align*}
   -\frac{4}{3}x + \frac{32}{3} &= \frac{3}{4}x + \frac{1}{4} \\
   x &= 5 \\
   y &= 4
   \end{align*}
   \]

   The coordinates of point \( E \) are \((5, 4)\).
d. Determine the height of parallelogram $ABCD$.

$$AE = \sqrt{(5 - 1)^2 + (4 - 1)^2}$$
$$= \sqrt{(4)^2 + (3)^2}$$
$$= 25$$

$$= 5$$

e. Determine the area of parallelogram $ABCD$.

$$A = bh$$
$$= 10 \times 5$$
$$= 50$$

The area of parallelogram $ABCD$ is 50 square units.

3. You determined earlier that any side of a parallelogram can be thought of as the base. Predict whether using a different side as the base will result in a different area of the parallelogram. Explain your reasoning.

Answers will vary.

No. I do not think using a different side as the base will matter because the size and the shape of the parallelogram do not change.
Grouping
Have students complete Questions 4 and 5 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Questions 4 and 5

- If line segment BC is thought of as the base of the parallelogram, how would you describe the location of the height?
- As you change the location of the base of the parallelogram, does the height of the parallelogram also change in location? Does it change in length?
- How is the location of the height determined?
- What is the point-slope equation for a line?
- How are the coordinates of the endpoints of the line segment representing the height determined?
- What are the equations for the two lines intersecting at the endpoint of the line segment representing the height?
- Is the area an approximation or an exact answer? Why?

Let’s consider your prediction.

4. Parallelogram ABCD is given on the coordinate plane. This time, let’s consider side BC as the base.

a. Let point E represent the intersection point of the height, AE, and the base. Determine the coordinates of point E.

Slope of base BC:
\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]
\[ m = \frac{0 - (-7)}{8 - 7} = \frac{7}{1} = 7 \]

Equation of height AE:
\[ (y - y_2) = m(x - x_1) \]
\[ (y - 1) = -\frac{1}{7}(x - 1) \]
\[ y = \frac{1}{7}x + \frac{8}{7} \]

Slope of height AE:
\[ m = -\frac{1}{7} \]

Equation of base BC:
\[ C(8, 0) \]
\[ (y - y_1) = m(x - x_1) \]
\[ (y - 0) = 7(x - 8) \]
\[ y = 7x - 56 \]

Solution of the system of equations:
\[ -\frac{1}{7}x + \frac{8}{7} = 7x - 56 \]
\[ 400 = 50x \]
\[ x = 8 \]
\[ y = 7(8) - 56 \]
\[ y = 0 \]

The coordinates of point E are (8, 0). These are the same coordinates as those for point C. The height can actually be represented by the line segment AC.
b. Determine the area of parallelogram $ABCD$.

$$AC = \sqrt{(8 - 1)^2 + (0 - 1)^2}$$
$$= \sqrt{7^2 + (-1)^2}$$
$$= \sqrt{49 + 1} = \sqrt{50}$$

$$A = bh$$
$$= (\sqrt{50})(\sqrt{50})$$
$$= 50$$

The area of parallelogram $ABCD$ is 50 square units.

5. Compare the area you calculated in Question 4, part (b) with the area you calculated in Question 2, part (e). Was your prediction in Question 3 correct? Explain why or why not.

Yes. My prediction was correct. The areas are the same no matter which side of the parallelogram you use as the base.
Problem 3
Students determine the area of a parallelogram on the coordinate plane using the rectangle method. The rectangle method involves drawing a rectangle around a parallelogram, creating right triangles whose areas can be subtracted from the area of the rectangle to determine the area of the parallelogram.

Grouping
Have students complete Questions 1 through 9 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Questions 1 and 2
• By drawing the rectangle through the vertices of the figure, what shape did you create on each side of the parallelogram?
• How can you determine the area of each of these shapes?

PROBLEM 3 Time for a Little Boxing
In the previous problem, you learned one method to calculate the area of a parallelogram.

1. Summarize the steps to calculate the area of a parallelogram using the method presented in Problem 2.
   • Calculate the length of a base using the Distance Formula.
   • Determine an equation for the base using the endpoints.
   • Determine an equation for the height using the slope and one point.
   • Determine the intersection of the lines for the base and height.
   • Calculate the length of the height using the Distance Formula.
   • Substitute the base and height into the area formula.

Now, let’s explore another method that you can use to calculate the area of a parallelogram. Consider parallelogram ABCD from Problem 2 Question 1.

2. Sketch a rectangle that passes through points A, B, C, D so that each side of the rectangle passes through one point of the parallelogram and all sides of the rectangle are either horizontal or vertical. Label the vertices of your rectangle as W, X, Y, and Z.

See coordinate plane.
Guiding Questions for Share Phase, Questions 3 through 9

• How can you use the coordinates of the parallelogram’s vertices to determine the coordinates of the rectangle’s vertices?
• How can you calculate the area of the rectangle?
• How can you calculate the area of each triangle?
• What advantages are there to using the rectangle method?

3. Determine the coordinates of W, X, Y, and Z. Explain how you calculated each coordinate.

Point W has the same x-coordinate as point A and the same y-coordinate as point D. The coordinates of point W are (1, 8).

Point X has the same x-coordinate as point C and the same y-coordinate as point D. The coordinates of point X are (8, 8).

Point Y has the same x-coordinate as point C and the same y-coordinate as point B. The coordinates of point Y are (8, −7).

Point Z has the same x-coordinate as point A and the same y-coordinate as point B. The coordinates of point Z are (1, −7).

4. Calculate the area of each figure. Show all your work.

a. rectangle WXYZ
The length of rectangle WXYZ is 7 units and its width is 15 units. So, the area of rectangle WXYZ is 15 \cdot 7, or 105 square units.

b. triangle ABZ
The base of triangle ABZ is 6 units and its height is 8 units. So, the area of triangle ABZ is \frac{1}{2}(6)(8), or 24 square units.

c. triangle BCY
The base of triangle BCY is 1 unit and its height is 7 units. So, the area of triangle BCY is \frac{1}{2}(1)(7), or 3.5 square units.

d. triangle CDX
The base of triangle CDX is 6 units and its height is 8 units. So, the area of triangle CDX is \frac{1}{2}(6)(8), or 24 square units.

e. triangle DAW
The base of triangle DAW is 1 unit and its height is 7 units. So, the area of triangle DAW is \frac{1}{2}(1)(7), or 3.5 square units.

f. parallelogram ABCD
The area of parallelogram ABCD is the area of rectangle WXYZ minus the areas of the four triangles.

Area of parallelogram ABCD = 105 + (3.5 + 24 + 24 + 3.5)
= 105 - 55
= 50

The area of parallelogram ABCD is 50 square units.
5. Compare the area you calculated in Question 4, part (f) to the area that you calculated in Problem 2. What do you notice?

The area of parallelogram $ABCD$ is 50 square units, regardless of the method that I used.

6. Summarize the steps to calculate the area of a parallelogram using the method presented in Problem 3.

- Sketch a rectangle that passes through the vertices of the parallelogram such that each side of the rectangle is horizontal or vertical.
- Calculate the area of the rectangle.
- Calculate the area of each triangle.
- Calculate the area of the parallelogram by subtracting the areas of the triangles from the area of the rectangle.

7. Which method for calculating the area of a parallelogram do you prefer? Why?

Answers will vary.

I prefer using the rectangle method. Bases and heights of the rectangle and each triangle are horizontal or vertical so the calculations are simpler. I am more likely to make a computational mistake using the distance formula.

8. Do you think the rectangle method will work to calculate the area of a triangle? Explain your reasoning.

Yes. I think the rectangle method will work for triangles as well. I can draw a rectangle around any triangle. The area of the triangle can be calculated by subtracting the areas of the three triangles from the area of the rectangle.
9. On the coordinate plane, draw your own triangle. Use the rectangle method to calculate the area of the triangle that you drew.

Answers will vary.

ELL Tip

Have each student create their own triangle with integer coordinates and use the rectangle method to calculate the area. Students should share the area of the triangle and the area of the rectangle. Ask students to compare their results and work together to make a conjecture about area of the triangle when compared to the area of the rectangle.
**Problem 4**

Students calculate the area, perimeter, and costs associated with a project involving a mass of land modeled by a parallelogram.

**Grouping**

Have students complete Questions 1 through 6 with a partner. Then have students share their responses as a class.

**Guiding Questions for Share Phase, Questions 1 through 6**

- Where did you place the axes on the coordinate plane? Why did you choose this location?
- What coordinates were used for the vertices of Tennessee?
- How did you determine the length of the base of Tennessee?
- How did you determine the height of Tennessee?
- How large is one square mile? One hundred square miles? One thousand square miles?
- Is the area exact or approximate? Why?
- What unit of measure is used to describe the area?
- Is the perimeter exact or approximate? Why?
- What unit of measure is used to describe the perimeter?
- How did you determine the population of Tennessee?

1. Transfer the state of Tennessee into the coordinate plane shown.

2. Suppose Tennessee had an outbreak of killer grasshoppers. One scientist says that it is necessary to spray insecticide over the entire state. Determine the approximate area that needs to be treated. Explain how you found your answer.

Tennessee closely resembles a parallelogram. Using the map key, I estimated the average length of the base of the parallelogram as \((440 + 320)/2 = 380\) miles and the height of the parallelogram as 110 miles. Then, I used the formula \(A = bh\) to determine the area of the parallelogram.

The area of Tennessee is approximately 41,800 square miles.

The approximate total area that needs to be treated is approximately 41,800 square miles.

- If each person living in Tennessee equally paid for the cost of spraying the entire state, how much would it cost each person?
- If each person living in Tennessee equally paid for the cost of spraying the perimeter of the state, how much would it cost each person?
3. Suppose one tank of insect spray costs $600, and the tank covers 1000 square miles. How much will this project cost? Show your work.
   First, calculate the number of tanks required for this project.
   \[ 41,800 \div 1000 = 41.8 \]
   This project requires 41.8 tanks of insect spray.
   Next, calculate the cost.
   \[ 41.8 \times 600 = 25,080 \]
   This project will cost $25,080.

4. A second scientist says it would only be necessary to spray the perimeter of the state. The type of spray needed to do this job is more concentrated and costs $2000 per tank. One tank of insect spray covers 100 linear miles. How much will this project cost? Show your work.
   Again, using the map key, I estimated the perimeter of Tennessee to be 440 + 320 + 130 + 230, or 1120 miles.
   Next, calculate the number of tanks required.
   \[ 1120 \div 100 = 11.2 \]
   Only spraying the perimeter requires 11.2 tanks.
   Finally, calculate the cost.
   \[ 11.2 \times 2000 = 22,400 \]
   This project will cost $22,400.

5. Which method of spraying is more cost efficient?
   Spraying the perimeter of the state is more cost efficient because it will save 25,080 – 22,400, or $2680.

6. If the population of Tennessee is approximately 153.9 people per square mile, how many people live in the state?
   Approximately \[ 41,800 \times 153.9 \], or 6,433,020 people live in Tennessee.
Problem 5
Students investigate again the effects of proportional and non-proportional changes on perimeter and area—this time with non-rectangular parallelograms. A parallelogram is drawn on the coordinate plane and when the height is doubled, the area is also doubled. Students manipulate the height of the parallelogram, redraw the parallelogram, and use a formula to calculate the area to verify it is twice the area of the given parallelogram.

Grouping
Have students complete Questions 1 through 3 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Questions 1 through 3
- Do you think that proportional and non-proportional changes will have similar effects on parallelograms as they did on triangles and rectangles? Explain.
- How would a proportional change to only one dimension of a parallelogram affect its perimeter and area?
- How would a non-proportional change to only one dimension of a parallelogram affect its perimeter and area?

PROBLEM 5  Reaching for New Heights, and Bases . . . One More Time
Recall that you have determined and described how proportional and non-proportional changes in the linear dimensions of rectangles and triangles affect their perimeter and area.

1. Determine and describe how subtracting 5 units from all sides of a parallelogram will affect the perimeter of the resulting parallelogram. Provide an example and explain your reasoning.
   Subtracting 5 units from all sides of a parallelogram will result in a parallelogram with perimeter that is 4\(\times\)5, or 20 units less.

   For example, consider a parallelogram with opposite side lengths of 12 centimeters and 9 centimeters. Subtracting 5 centimeters from each side results in a parallelogram with a perimeter that is 4\(\times\)5, or 20 centimeters, less than the perimeter of the original parallelogram, 42 centimeters.

   Perimeter of Original Parallelogram (cm):
   \[2(12) + 2(9) = 42\]
   Perimeter of Resulting Triangle (cm):
   \[2(7) + 2(4) = 22\]
   Difference between Resulting Perimeter and Original Perimeter (cm):
   \[42 - 22 = 20\]

2. Describe how multiplying the base and height of a parallelogram by a factor of 10 will affect the area of the resulting parallelogram. Provide an example, determine its area, and explain your reasoning.
   Multiplying the base and height of a parallelogram by a factor of 10 will result in a parallelogram with an area that is 10\(\times\)10, or 100 times greater than the original parallelogram.

   For example, consider a parallelogram with base 3 inches and height 4 inches. Multiplying the base and height by 10 results in a base of 30 and a height of 40 inches. The area of the resulting parallelogram, 1200 square inches, is 100 times greater than the area of the original parallelogram, 12 square inches.

   Area of Original Parallelogram (square inches):
   \[3(4) = 12\]
   Area of Resulting Parallelogram (square inches):
   \[30(40) = 1200\]
   Ratio between Resulting Area and Original Area (square inches):
   \[\frac{1200}{12} = 100\]
3. Parallelogram $ABCD$ is given. Double the area of parallelogram $ABCD$ by manipulating the height. Label the image, identify the coordinates of the new point(s), and determine the area.

If points $B$ and $C$ are manipulated, the new points are $B' (-3, 14)$ and $C' (5, 14)$. If points $A$ and $D$ are manipulated, the new points are $A' (-7, 5)$ and $D' (1, 5)$.

$$A = bh$$

$A = (8)(6)$

$A = 48$

The area of the new parallelogram is 48 square units.

Be prepared to share your solutions and methods.
Check for Students’ Understanding

Four points and their coordinates are given.

1. Determine if the quadrilateral is a parallelogram. Show your work.
   Yes. Quadrilateral $ABCD$ is a parallelogram.

   Slope of line segment $AB$:
   \[ m = \frac{y_2 - y_1}{x_2 - x_1} \]
   \[ m = \frac{3 - (-8)}{-6 - (-10)} = \frac{11}{4} \]

   Slope of line segment $BC$:
   \[ m = \frac{y_2 - y_1}{x_2 - x_1} \]
   \[ m = \frac{-8 - (-8)}{-10 - 5} = \frac{0}{-15} = 0 \]

   Slope of line segment $CD$:
   \[ m = \frac{y_2 - y_1}{x_2 - x_1} \]
   \[ m = \frac{3 - (-8)}{9 - 5} = \frac{11}{4} \]

   Slope of line segment $AD$:
   \[ m = \frac{y_2 - y_1}{x_2 - x_1} \]
   \[ m = \frac{3 - 3}{-6 - 9} = \frac{0}{-15} = 0 \]
Leavin’ on a Jet Plane
Area and Perimeter of Trapezoids on the Coordinate Plane

LEARNING GOALS
In this lesson, you will:
• Determine the perimeter and the area of trapezoids and hexagons on a coordinate plane.
• Determine and describe how proportional and non-proportional changes in the linear dimensions of a trapezoid affect its perimeter and area.

KEY TERMS
• bases of a trapezoid
• legs of a trapezoid

ESSENTIAL IDEAS
• Rigid motion is used to change the position of trapezoids on the coordinate plane.
• Rigid motion is used to determine the perimeter of parallelograms.
• The Pythagorean Theorem is used to determine the perimeter of trapezoids.
• The rectangle method is used to determine the area of trapezoids on the coordinate plane.
• Velocity-Time graphs are used to model situations.
• When the dimensions of a plane figure change proportionally by a factor of $k$, its perimeter changes by a factor of $k$, and its area changes by a factor of $k^2$.
• When the dimensions of a plane figure change non-proportionally, its perimeter and area increase or decrease non-proportionally.

TEXAS ESSENTIAL KNOWLEDGE AND SKILLS FOR MATHEMATICS
(3) Coordinate and transformational geometry. The student uses the process skills to generate and describe rigid transformations (translation, reflection, and rotation) and non-rigid transformations (dilations that preserve similarity and reductions and enlargements that do not preserve similarity). The student is expected to:
• (B) determine the image or pre-image of a given two-dimensional figure under a composition of rigid transformations, a composition of non-rigid transformations, and a composition of both, including dilations where the center can be any point in the plane.
Overview

Rigid motion is used to explore the perimeter of trapezoids on the coordinate plane. Students begin the lesson by performing a translation on a trapezoid. Using the Pythagorean Theorem, they are able to determine the perimeter of a trapezoid. Next, they divide a parallelogram into two congruent trapezoids to develop a formula to compute the area of any trapezoid. Students then use the rectangle method to determine the area of a trapezoid on the coordinate plane. Scenarios using velocity-time graphs are used to compute time and distance related to the problem situation. Students also investigate how proportional and non-proportional changes to the linear dimensions of a plane figure affect its perimeter and area.
Warm Up

Given trapezoid $ABCD$ with $AB \perp BC$ and $DC \perp BC$.

1. Describe the relationship between lines segments $AB$ and $DC$ in trapezoid $ABCD$.
   Line segments $AB$ and $DC$ are parallel to each other because they are both perpendicular to line segment $BC$.

2. What information is needed to determine the perimeter of trapezoid $ABCD$?
   The information needed to determine the perimeter of trapezoid $ABCD$ are the lengths of line segments $AB$, $BC$, $CD$, and $AD$.

3. Considering the coordinates of each vertex, which side length is not obvious and how can the length be determined?
   The length of line segment $AD$ is not obvious. The Distance Formula or the Pythagorean Theorem can be used to determine the length of line segment $AD$.

4. Determine the perimeter of trapezoid $ABCD$. Estimate the value of radicals to the nearest tenth.
   
   \[ a^2 + b^2 = c^2 \]
   
   \[ (5)^2 + (8)^2 = (c)^2 \]
   
   \[ c^2 = 25 + 64 \]
   
   \[ c^2 = 89 \]
   
   \[ c = \sqrt{89} \approx 9.4 \]

   Perimeter: $7 + 8 + 2 + 9.4 = 26.4$
   
   The approximate perimeter of trapezoid $ABCD$ is 26.4 units.
How can you make a building withstand an earthquake? The ancient Incas figured out a way—by making trapezoidal doors and windows.

The Inca Empire expanded along the South American coast—an area that experiences a lot of earthquakes—from the 12th through the late 15th century.

One of the most famous of Inca ruins is Machu Picchu in Peru. There you can see the trapezoidal doors and windows—tilting inward from top to bottom to better withstand the seismic activity.
Problem 1

Students graph a trapezoid which extends into each of the four quadrants. Without using the Distance Formula, they devise and implement a strategy to determine the perimeter of the trapezoid.

Grouping

Have students complete Questions 1 and 2 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Questions 1 and 2

- Is the quadrilateral a parallelogram? Why or why not?
- How would you describe the orientation of this quadrilateral?
- Are any sides of the quadrilateral parallel to each other? How do you know?
- Are any sides of the quadrilateral perpendicular to each other? How do you know?
- How do the slopes of the line segments compare to each other?
- Which sides of quadrilateral $ABCD$ are parallel to each other?

PROBLEM Well, It’s the Same, But It’s Also Different!

So far, you have determined the perimeter and the area of parallelograms—including rectangles and squares. Now, you will move on to trapezoids.

1. Plot each point in the coordinate plane shown:
   - $A(-5, 4)$
   - $B(-5, -4)$
   - $C(6, -4)$
   - $D(0, 4)$

   Then, connect the points in alphabetical order.
   See coordinate plane.

2. Explain how you know that the quadrilateral you graphed is a trapezoid.
   I know that it is a trapezoid because the quadrilateral only has one pair of parallel sides.
The trapezoid is unique in the quadrilateral family because it is a quadrilateral that has exactly one pair of parallel sides. The parallel sides are known as the bases of the trapezoid, while the non-parallel sides are called the legs of the trapezoid.

3. Using the trapezoid you graphed, identify:
   a. the bases.
   The bases of trapezoid $ABCD$ are $AD$ and $BC$.
   b. the legs.
   The legs of trapezoid $ABCD$ are $AB$ and $DC$.

4. Analyze trapezoid $ABCD$ that you graphed on the coordinate plane.
   a. Describe how you can determine the perimeter of trapezoid $ABCD$ without using the Distance Formula.
   To determine the perimeter of trapezoid $ABCD$ without using the Distance Formula, I can translate the figure 5 units to the right, then 4 units up. By doing this, leg $AB$ will be on the $y$-axis, vertex $B$ will be at the origin, and base $BC$ will be on the $x$-axis. Three of the sides are horizontal or vertical so I can subtract coordinates to calculate their length. I can create a right triangle and use the Pythagorean Theorem to determine the length of $DC$.

b. Determine the perimeter of trapezoid $ABCD$ using the strategy you described in part (a). First, perform a transformation of trapezoid $ABCD$ on the coordinate plane and then calculate the perimeter of the image.
   See coordinate plane.
   $A'B' = 8 - 0 = 8$ units
   $B'C' = 11 - 0 = 11$ units
   $A'D' = 5 - 0 = 5$ units
   
   $$(C'D')^2 = 8^2 + 6^2$$
   $$(C'D')^2 = 64 + 36$$
   $$(C'D')^2 = 100$$
   $C'D' = \sqrt{100}$
   $C'D' = 10$

   $$P = A'B' + B'C' + A'D' + C'D'$$
   $$= 8 + 11 + 5 + 10$$
   $$= 34$$
   The perimeter of trapezoid $ABCD$ is 34 units.

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**Grouping**
Ask a student to read the definitions and information aloud and complete Question 3 as a class.

**Guiding Questions for Discuss Phase, Question 3**
- What is the difference between the leg of a trapezoid and the base of a trapezoid?
- Do all trapezoids have exactly two legs?
- Do all trapezoids have exactly two bases?
- Do you suppose the bases of a trapezoid could be congruent to each other?
- Do you suppose the legs of a trapezoid could be congruent to each other?

**Grouping**
Have students complete Question 4 with a partner. Then have students share their responses as a class.

**Guiding Questions for Share Phase, Question 4**
- Is the Distance Formula needed to calculate the length of each of the four sides of the trapezoid?
- What are other methods that can be used to determine the length of each of the four sides of the trapezoid?
- Is transforming the trapezoid helpful in determining the perimeter? Why or why not?
- What unit of measure is associated with the perimeter of the trapezoid?
Problem 2
Students divide a parallelogram into two trapezoids to develop the formula for the area of a trapezoid. Given four coordinates, they graph a trapezoid on a coordinate plane and use the formula to determine the area of the trapezoid.

Grouping
Have students complete Questions 1 and 2 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Questions 1 and 2
• After you have located and labeled $b_1$ and $b_2$, what algebraic expression represents the length of the base of the parallelogram?
• Can $\frac{b_1 + b_2}{2}$ be written as $\frac{1}{2}(b_1 + b_2)$? Why or why not?

PROBLEM 2 Using What You Know
So, what similarities are there between determining the area of a parallelogram and determining the area of a trapezoid?

Recall that the formula for the area of a parallelogram is $A = bh$, where $b$ represents the base and $h$ represents the height. As you know, a parallelogram has both pairs of opposite sides parallel. But what happens if you divide the parallelogram into two congruent trapezoids?

1. Analyze parallelogram $FGHJ$ on the coordinate plane.

   ![Graph of parallelogram](image)

   a. Divide parallelogram $FGHJ$ into two congruent trapezoids.
      
      See coordinate plane.

   b. Label the two vertices that make up the two congruent trapezoids.
      
      See coordinate plane.

   c. Label the bases that are congruent to each other. Label one pair of bases $b_1$ and the other pair $b_2$.
      
      See coordinate plane.

   d. Now write a formula for the area of the entire geometric figure. Make sure you use the bases you labeled and do not forget the height.
      
      Area of a parallelogram is $A = bh$.
      
      For the area of this geometric figure, I would substitute the $b$ with the two bases.
      
      \[ A = (b_1 + b_2)h \]

   e. Now write the formula for the area for half of the entire figure.
      
      \[ A = \left(\frac{b_1 + b_2}{2}\right)h \]
Grouping
Have students complete Questions 3 and 4 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Questions 3 and 4
- Did you transform the trapezoid before determining the area? Why or why not?
- How is the location of the height determined?
- Is the area an approximation or an exact answer?
- What unit of measure is associated with the area of the trapezoid?

2. What can you conclude about the area formula of a parallelogram and the area formula of a trapezoid? Why do you think this connection exists?

The area formula for a trapezoid is one half of the area formula of a parallelogram because I know that a parallelogram can be divided into two congruent trapezoids.

3. Plot each point on the coordinate plane shown:
- Q(−2, −2)
- R(5, −2)
- S(5, 2)
- T(1, 2)

Then, connect the points in alphabetical order.

4. Determine the area of trapezoid QRST. Describe the strategy or strategies you used to determine your answer.

I translated trapezoid QRST so that point Q was at the origin by translating it 2 units to the right and 2 units up.

\[ A = \frac{b_1 + b_2}{2} \times h \]
\[ A = \frac{4 + 7}{2} \times 4 \]
\[ A = \frac{11}{2} \times 4 \]
\[ A = 22 \]

The area of trapezoid QRST is 22 square units.
**Problem 3**
Students use the rectangle method to determine the area of a trapezoid on the coordinate plane.

**Grouping**
Have students complete Questions 1 through 6 with a partner. Then have students share their responses as a class.

**Guiding Questions for Share Phase, Questions 1 through 6**
- By drawing the rectangle through the vertices of the figure, what shape did you create on each side of the trapezoid?
- How can you determine the area of each of these shapes?
- How can you use the coordinates of the trapezoid’s vertices to determine the coordinates of the rectangle’s vertices?
- How can you calculate the area of the rectangle?
- How can you calculate the area of each triangle?
- Can you draw a trapezoid on the coordinate plane whose area would be difficult to determine using the rectangle method?

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**PROBLEM Box Up That Trapezoid**

In the previous lesson, you learned a method to calculate the area of a parallelogram using a rectangle. Can the same method be used to calculate the area of a trapezoid?

1. Summarize the steps to calculate the area of a parallelogram using the rectangle method.
   - Sketch a rectangle that passes through the vertices of the parallelogram such that each side of the rectangle is horizontal or vertical.
   - Calculate the area of the rectangle.
   - Calculate the area of each triangle.
   - Calculate the area of the parallelogram by subtracting the areas of the triangles from the area of the rectangle.

Consider trapezoid $ABCD$.

2. Sketch a rectangle that passes through points $A$, $B$, $C$, $D$ so that each side of the rectangle passes through one point of the trapezoid and all sides of the rectangle are either horizontal or vertical. Label the vertices of your rectangle as $W$, $X$, $Y$, and $Z$.

See coordinate plane.
3. Determine the coordinates of W, X, Y, and Z. Explain how you calculated each coordinate.

Point W has the same x-coordinate as point A and the same y-coordinate as point D. The coordinates of point W are (−4, 4).

Point X has the same x-coordinate as point C and the same y-coordinate as point D. The coordinates of point X are (5, 4).

Point Y has the same x-coordinate as point C and the same y-coordinate as point B. The coordinates of point Y are (5, −5).

Point Z has the same x-coordinate as point A and the same y-coordinate as point B. The coordinates of point Z are (−4, −5).

4. Calculate the area of trapezoid ABCD using the rectangle method. Show all your work.

The length of rectangle WXYZ is 9 units and its width is 9 units. So, the area of rectangle WXYZ is $9 \times 9$, or 81 square units.

The base of triangle ABX is 5 units and its height is 2 units. So, the area of triangle ABZ is $\frac{1}{2}(5)(2)$, or 5 square units.

The base of triangle BCY is 8 units and its height is 7 units. So, the area of triangle BCY is $\frac{1}{2}(8)(7)$, or 28 square units.

The base of triangle CDZ is 1 unit and its height is 6 units. So, the area of triangle CDX is $\frac{1}{2}(1)(6)$, or 3 square units.

The base of triangle DAW is 1 unit and its height is 7 units. So, the area of triangle DAW is $\frac{1}{2}(1)(7)$, or 3.5 square units.

The area of parallelogram ABCD is the area of rectangle WXYZ minus the areas of the four triangles.

Area of parallelogram ABCD $= 81 - (5 + 28 + 3 + 6)$

$= 81 - 42$

$= 39$

The area of parallelogram ABCD is 39 square units.

5. Do you think the rectangle method will work to calculate the area of any trapezoid? Explain your reasoning.

Yes. I think the rectangle method will work for any trapezoid. I can draw a rectangle around any trapezoid.
6. On the coordinate plane, draw your own trapezoid. Use the rectangle method to calculate the area of the trapezoid that you drew.

Answers will vary.
Problem 4
Students investigate again the effects of proportional and non-proportional changes on perimeter and area—this time with trapezoids.

Grouping
Have students complete the problem with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Problem 4
• What plane figures can compose a trapezoid?
• How do proportional changes to the dimensions of a trapezoid affect the plane shapes (triangles, rectangle) that compose it?

PROBLEM 4  Reaching for New Heights, and Bases . . . One Last Time
Recall that you have determined and described how proportional changes in the linear dimensions of rectangles, triangles, and parallelograms affect their perimeter and area. You can apply that knowledge to trapezoids.

1. Describe how multiplying the bases and height of a trapezoid by a factor of \( \frac{1}{4} \) will affect the area of the resulting trapezoid. Provide an example, determine its area, and explain your reasoning.

Multiplying the bases and height of a trapezoid by a factor of \( \frac{1}{4} \) will result in a trapezoid whose area is \( \frac{1}{4} \times \frac{1}{4} \), or \( \frac{1}{16} \), of the original trapezoid.

For example, consider a trapezoid with bases 12 inches and 20 inches, and height 8 inches. Multiplying the bases and height by \( \frac{1}{4} \) results in bases of 3 inches and 5 inches, and a height of 2 inches. The area of the resulting trapezoid, 24 square inches, is \( \frac{1}{16} \) of the original area, 384 square inches.

Area of Original Trapezoid (square inches):
\[
\frac{1}{2}(12 + 20)8 = 128
\]

Area of Resulting Trapezoid (square inches):
\[
\frac{1}{2}(3 + 5)2 = 8
\]

Ratio between Resulting Area and Original Area (square inches):
\[
\frac{8}{128} = \frac{1}{16}
\]
Problem 5
Scenarios are used in which velocity-time graphs are given. The area of the region under the line or curve on the graph represents distance. The graphs used are associated with the speed of a car and the speed of a jet.

Grouping
Have students complete Questions 1 and 2 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Questions 1 and 2
• If a vertical line segment is drawn where the time is 2.5 hours, what geometric figure is formed?
• How is the area of the geometric figure determined?
• Is the area of the geometric figure the same as the distance the car has traveled? Why or why not?

PROBLEM 5  Jets and Trapezoids!

The graph shows the constant speed of a car on the highway over the course of 2.5 hours.

1. Describe how you could calculate the distance the car traveled in 2.5 hours using what you know about area.
   I can draw a vertical line segment at 2.5 hours to form a rectangle on the graph. Because distance = rate × time, I can calculate the area of the rectangle (b × h) to calculate the total distance traveled.

2. How far did the car travel in 2.5 hours?
   The car traveled 60 × 2.5, or 150, miles.
Grouping

Have students complete Questions 3 through 6 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Questions 3 through 6

• How is determining the distance the jet traveled similar to determining the distance the car traveled?
• How is determining the distance the jet traveled different than determining the distance the car traveled?

3. How can you use the graph to determine the distance the jet has traveled in 25 minutes?
   I can draw a vertical line segment at 25 minutes. The area of the region enclosed by the line segments represents how far the jet has traveled in 25 minutes.

4. What shape is the region on the graph enclosed by the line segments?
   The region on the graph enclosed by the line segments is a trapezoid.

5. Determine the distance the jet has traveled in 25 minutes. Show your work.
   \[ A = \frac{1}{2} (b_1 + b_2)h \]
   \[ = \frac{1}{2} \left( \frac{25}{60} + \frac{20}{60} \right) \times 600 \]
   \[ = \frac{1}{2} \left( \frac{45}{60} \right) \times 600 \]
   \[ = \frac{1}{2} (45)(10) \]
   \[ = 225 \]
   The jet has traveled 225 miles in 25 minutes.

6. Determine the distance the jet has traveled in 5 minutes. Show your work.
   \[ A = \frac{1}{2}bh \]
   \[ = \frac{1}{2} \left( \frac{5}{60} \right) \times 600 \]
   \[ = \frac{1}{2} \times 25 \]
   \[ = 25 \]
   The jet has traveled 25 miles in 5 minutes.

Be prepared to share your solutions and methods.
Check for Students’ Understanding

1. Draw a velocity-time graph describing the ascent of a passenger jet using the following information.
   - The jet took 7 minutes to reach a top speed of 600 miles per hour.
   - The jet continued to travel at a constant speed of 600 miles per hour.
   - The jet left the airport 4 hours ago.

![Velocity-time graph]

2. How many miles has the jet traveled?

\[
A = \frac{1}{2}(b_1 + b_2)h \\
= \frac{1}{2} \left( \frac{240}{60} + \frac{223}{60} \right) \times 600 \\
= \frac{1}{2} \left( \frac{463}{60} \right) \times 600 = \frac{1}{2} (463)(10) = 2315
\]

The jet has traveled 2315 miles in 4 hours.
Composite Figures on the Coordinate Plane

Area and Perimeter of Composite Figures on the Coordinate Plane

LEARNING GOALS

In this lesson, you will:

- Determine the perimeters and the areas of composite figures on a coordinate plane.
- Connect transformations of geometric figures with number sense and operations.
- Determine the perimeters and the areas of composite figures using transformations.

KEY TERM

- composite figures

ESSENTIAL IDEAS

- A composite figure is a figure that is formed by combining different shapes.
- The area of a composite figure is determined by dividing the figure into familiar shapes and using the area formulas associated with those shapes.
- The rectangle method can be used to determine the area of composite figures on the coordinate plane.
- The Pythagorean Theorem is used to determine the perimeter of composite figures on the coordinate plane.

TEXAS ESSENTIAL KNOWLEDGE AND SKILLS FOR MATHEMATICS

(3) Coordinate and transformational geometry. The student uses the process skills to generate and describe rigid transformations (translation, reflection, and rotation) and non-rigid transformations (dilations that preserve similarity and reductions and enlargements that do not preserve similarity). The student is expected to:

(B) determine the image or pre-image of a given two-dimensional figure under a composition of rigid transformations, a composition of non-rigid transformations, and a composition of both, including dilations where the center can be any point in the plane.
(11) Two-dimensional and three-dimensional figures. The student uses the process skills in the application of formulas to determine measures of two- and three-dimensional figures. The student is expected to:

(B) determine the area of composite two-dimensional figures comprised of a combination of triangles, parallelograms, trapezoids, kites, regular polygons, or sectors of circles to solve problems using appropriate units of measure

Overview

Students determine the area and perimeter of composite figures drawn on the coordinate plane. Students divide the composite figures different ways into various known polygons to compute unknown measurements. Students use the rectangle method to determine the area of a composite figure.
Warm Up

1. Divide this region into familiar polygons by connecting vertices to form one or more line segments.

![Diagram with vertices A(0, 0), B(0, -12), C(20, -12), D(20, 5), E(9, 5), F(9, -4)]

2. How many line segments did you use to divide the region into familiar polygons?
   Answers will vary.
   I used one line segment to divide the region into familiar polygons.

3. Which polygons were formed by the line segment(s)?
   Answers will vary.
   The region was divided into a rectangle and a trapezoid.

4. Did your classmates divide the region the same way?
   No. There are several different ways to divide the region into familiar polygons.

5. If you were interested in determining the area of the region, which method for dividing the polygon would you use? Why?
   Answers will vary.
   I would prefer to divide the region into as few polygons as possible to eliminate additional steps in the solution path.
Composite Figures on the Coordinate Plane

Area and Perimeter of Composite Figures on the Coordinate Plane

LEARNING GOALS
In this lesson, you will:

- Determine the perimeters and the areas of composite figures on a coordinate plane.
- Connect transformations of geometric figures with number sense and operations.
- Determine the perimeters and the areas of composite figures using transformations.

Did you ever think about street names? How does a city or town decide what to name their streets?

Some street names seem to be very popular. In the United States, almost every town has a Main Street. But in France, there is literally a Victor Hugo Street in every town!

Victor Hugo was a French writer. He is best known for writing the novels Les Misérables and Notre-Dame de Paris, better known as The Hunchback of Notre Dame in English.

If you were in charge of naming the streets in your town, what names would you choose? Would you honor any people with their own streets?
Problem 1
Students are given the graph of a composite figure and asked to determine the perimeter and area of the figure. Students will draw line segments on the figure to divide it into familiar polygons and work with those polygons. They do this activity twice, dividing the composite figure two different ways and conclude the area and perimeter remain unaltered.

Grouping
• Ask a student to read the definition and information aloud. Discuss as a class.
• Have students complete Questions 1 through 4 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Questions 1 through 4
• How would you describe the orientation of this composite figure on the coordinate plane?
• How many sides are on this composite figure?
• What familiar polygons did you divide the composite figure into?
• Is the Distance Formula needed to calculate the length of any sides of the composite figure? Why or why not?

PROBLEM Breakin’ It Down
Now that you have determined the perimeters and the areas of various quadrilaterals and triangles, you can use this knowledge to determine the perimeters and the areas of composite figures. A composite figure is a figure that is formed by combining different shapes. To determine the area of a composite figure, divide it into basic shapes.

1. A composite figure is graphed on the coordinate plane shown.

Determine the perimeter of the composite figure. Round to the nearest tenth if necessary.

Calculate the length of each horizontal or vertical segment.

- \( AB = 6 - (-2) = 8 \)
- \( FG = 3 - (-3) = 6 \)
- \( CD = 4 - (-2) = 6 \)
- \( HJ = 6 - (-12) = 18 \)
- \( DE = 7 - 3 = 4 \)
- \( JA = 10 - 3 = 7 \)
- \( EF = -2 - (-8) = 6 \)

Calculate the lengths of the remaining segments.

- \( BC^2 = 6^2 + 10^2 \)
- \( GH^2 = 7^2 + 4^2 \)
- \( BC^2 = 36 + 100 \)
- \( GH^2 = 49 + 16 \)
- \( BC^2 = 136 \)
- \( GH^2 = 65 \)
- \( BC = \sqrt{136} \)
- \( GH = \sqrt{65} \)

\[ P = AB + BC + CD + DE + EF + FG + GH + HJ + JA \]
\[ = 8 + \sqrt{136} + 6 + 4 + 6 + 6 + \sqrt{65} + 18 + 7 \]
\[ = 74.7 \text{ units} \]

The perimeter of this figure is approximately 74.7 units.

• Is the Pythagorean Theorem needed to calculate the length of any sides of the composite figure? Why or why not?
• Is there more than one way to divide this composite figure into familiar polygons? How?
• Would transforming the composite figure be helpful? Why or why not?
2. Draw line segments on the composite figure to divide the figure. Determine the area of the composite figure. Round to the nearest tenth if necessary.

I divided the figure into two triangles, a square, and a rectangle.

Area of left triangle $= \frac{1}{2}(10)(6) = 30$

Area of right triangle $= \frac{1}{2}(7)(4) = 14$

Area of rectangle $= 14(7) = 98$

Area of square $= 6^2 = 36$

$A = 30 + 14 + 98 + 36 = 178$ square units

The area of this figure is 178 square units.

3. Draw line segments on the composite figure to divide the figure differently from how you divided it in Question 2. Determine the area of the composite figure. Round to the nearest tenth if necessary.

I drew a large rectangle around the entire figure. I divided the top region that was not part of the original figure into a triangle and a rectangle. I divided the bottom region that was not part of the original figure into a rectangle and a trapezoid.

Area of large rectangle $= 18(17) = 306$

Area of top triangle $= \frac{1}{2}(10)(6) = 30$

Area of top rectangle $= 10(2) = 20$

Area of bottom rectangle $= 10(4) = 40$

Area of bottom trapezoid $= \frac{1}{2}(6 + 13)(4) = 38$

Area of figure $= 306 - (30 + 20 + 40 + 38) = 178$

The area of the figure is 178 square units.
Problem 2
Students analyze a representation of France mapped onto a coordinate plane and answer questions associated with the problem situation.

Grouping
Have students complete Questions 1 through 4 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Questions 1 through 4
- What method did you use to compute the approximate length of the coastline?
- What method did you use to compute the approximate area?
- How was the population of France determined? Did you use a conversion? How?

4. How does the area in Question 2 compare to the area in Question 3? Explain your reasoning.

The areas of the composite figure in Question 2 and Question 3 are equal because dividing the composite figure differently does not alter the shape or the size of the figure.
2. Which of the following statements is true?
   - The coastline of France is greater than 5000 kilometers.
   - The coastline of France is less than 5000 kilometers.
   - The coastline of France is approximately 5000 kilometers,
     \[ \text{Calculations will vary depending on the hexagon drawn in Question 1.} \]
     The coastline of France is approximately 3427 kilometers, so the coastline of France is less than 5000 kilometers.

3. Which of the following statements is true?
   - The area of France is greater than 1,000,000 square kilometers.
   - The area of France is less than 1,000,000 square kilometers.
   - The area of France is approximately 1,000,000 square kilometers.
     The area of France is approximately 547,000 square kilometers, so the area of France is less than 1,000,000 kilometers.

4. If the population of France is approximately 118.4 people per square mile, how many people live in the country of France?
   - Approximately 547,000 $\times$ 118.4, or 65,000,000 people live in the country of France.
**Talk the Talk**

Students draw line segments on a composite figure drawn on a coordinate plane to divide the figure into familiar polygons two different ways and compute the area using each method.

**Grouping**

Have students complete the Talk the Talk with a partner. Then have students share their responses as a class.

**ELL Tip**

There are many ways the composite figure can be divided into shapes. Have students present at least four different ways and give reasons which way they find preferable. They should support their opinions by being able to explain how they calculated the area in each solution. Remind students that methods can involve addition and/or subtraction.

Answers will vary.

I extended the lines to form a square. The area of the original figure is equal to the area of the square minus the areas of the two triangles.

The area of the square is \(30^2\), or 900 square units.

The area of each triangle is \(\frac{1}{2}(10)(10)\), or 50 square units.

The area of the figure is \(900 - (50 + 50)\), or 800 square units.

I could also draw two vertical segments to create two congruent trapezoids and a rectangle.

The area of each trapezoid is \(\frac{1}{2}(30 + 20)(10)\), or 250 square units.

The area of the rectangle is \(10(30)\), or 300 square units.

The area of the figure is \(250 + 250 + 300\), or 800 square units.

The area is the same using each method.

Be prepared to share your solutions and methods.
Check for Students’ Understanding

1. Divide this region into familiar polygons by connecting vertices to form one or more line segments.

   ![Diagram of the region with vertices A(0, 0), B(0, -12), C(20, -12), D(20, 5), E(9, 5), and F(9, -4).]

2. Determine the perimeter of this composite figure.

   ![Diagram of the composite figure with dimensions a=9, b=11, c=4, and d=9.8.]

   
   
   
   \[ a^2 + b^2 = c^2 \]

   \[ (9)^2 + (4)^2 = (AF)^2 \]

   \[ (AF)^2 = 81 + 16 \]

   \[ AF = \sqrt{97} \approx 9.8 \]

   
   
   
   
   
   
   
   
   
   \[ 9 + 11 + 17 + 11 + 9 + 9.8 + 4 + 8 = 78.8 \]

   The approximate perimeter is 78.8 units.
3. Determine the area of this composite figure.

Area of Trapezoid:

\[ A = \frac{1}{2}(b_1 + b_2)h \]

\[ = \frac{1}{2}(12 + 8)9 \]

\[ = \frac{1}{2}(20)9 = 90 \]

Area of Rectangle:

\[ A = bh \]

\[ = (11)(17) = 187 \]

The area of the composite figure is 90 + 187 = 277 square units.
Chapter 3 Summary

KEY TERMS
- bases of a trapezoid (3.4)
- legs of a trapezoid (3.4)
- composite figure (3.5)

3.1 Determining the Perimeter and Area of Rectangles and Squares on the Coordinate Plane

The perimeter or area of a rectangle can be calculated using the distance formula or by counting the units of the figure on the coordinate plane. When using the counting method, the units of the x-axis and y-axis must be considered to count accurately.

Example

Determine the perimeter and area of rectangle JKLM.

The coordinates for the vertices of rectangle JKLM are J(−120, 250), K(60, 250), L(60, −50), and M(−120, −50).

Because the sides of the rectangle lie on grid lines, subtraction can be used to determine the lengths.

\[ JK = 60 - (-120) = 180 \]
\[ KL = 250 - (-50) = 300 \]
\[ A = bh = 180(300) = 54,000 \]

\[ P = JK + KL + LM + JM = 180 + 300 + 180 + 300 = 960 \]

The area of rectangle JKLM is 54,000 square units.

The perimeter of rectangle JKLM is 960 units.
3.1 Using Transformations to Determine the Perimeter and Area of Geometric Figures

If a rigid motion is performed on a geometric figure, not only are the pre-image and the image congruent, but both the perimeter and area of the pre-image and the image are equal. Knowing this makes solving problems with geometric figures more efficient.

Example

Determine the perimeter and area of rectangle $ABCD$.

The vertices of rectangle $ABCD$ are $A(-20, 80), B(60, 80), C(60, 60),$ and $D(-20, 60)$. To translate point $D$ to the origin, translate $ABCD$ to the right 20 units and down 60 units. The vertices of rectangle $A'B'C'D'$ are $A'(0, 20), B'(80, 20), C'(80, 0),$ and $D'(0, 0)$.

Because the sides of the rectangle lie on grid lines, subtraction can be used to determine the lengths.

- $A'D' = 20 - 0 = 20$
- $C'D' = 80 - 0 = 80$
- $P = A'B' + B'C' + C'D' + A'D'$
  $= 80 + 20 + 80 + 20$
  $= 200$

The perimeter of rectangle $A'B'C'D'$ and, therefore, the perimeter of rectangle $ABCD$, is 200 units.

- $A = bh$
  $= 20(80)$
  $= 1600$

The area of rectangle $A'B'C'D'$ and, therefore, the area of rectangle $ABCD$, is 1600 square units.
Determining the Effect of Proportional and Non-Proportional Change on Perimeter and Area of a Rectangle

Proportional Change

- The perimeter of a rectangle with base $b$ and height $h$ will change by a factor of $k$, given that its original dimensions are multiplied by a factor of $k$.
- The area of a rectangle with base $b$ and height $h$ will change by a factor of $k^2$, given that its original dimensions are multiplied by a factor of $k$.

Example

<table>
<thead>
<tr>
<th></th>
<th>Original Rectangle</th>
<th>Rectangle Formed by Doubling Dimensions</th>
<th>Rectangle Formed by Tripling Dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear Dimensions</td>
<td>$b = 5$ in. $h = 4$ in.</td>
<td>$b = 10$ in. $h = 8$ in.</td>
<td>$b = 15$ in. $h = 12$ in.</td>
</tr>
<tr>
<td>Perimeter (in.)</td>
<td>$2(5 + 4) = 18$</td>
<td>$2(10 + 8) = 36$</td>
<td>$2(15 + 12) = 54$</td>
</tr>
<tr>
<td>Area (in.$^2$)</td>
<td>$5(4) = 20$</td>
<td>$10(8) = 80$</td>
<td>$15(12) = 180$</td>
</tr>
</tbody>
</table>

Non-Proportional Change

- The perimeter of a rectangle whose dimensions change non-proportionally by $x$ (adding $x$ to or subtracting $x$ from the dimensions) will change by a factor of $4x$.
- When the dimensions of a rectangle change non-proportionally, the resulting area changes, but there is not a clear pattern of increase or decrease.

Example

<table>
<thead>
<tr>
<th></th>
<th>Original Rectangle</th>
<th>Rectangle Formed by Adding 2 Inches to Dimensions</th>
<th>Rectangle Formed by Adding 3 Inches to Dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear Dimensions</td>
<td>$b = 5$ in. $h = 4$ in.</td>
<td>$b = 7$ in. $h = 6$ in.</td>
<td>$b = 8$ in. $h = 7$ in.</td>
</tr>
<tr>
<td>Perimeter (in.)</td>
<td>$2(5 + 4) = 18$</td>
<td>$2(7 + 6) = 26$</td>
<td>$2(8 + 7) = 54$</td>
</tr>
<tr>
<td>Area (in.$^2$)</td>
<td>$5(4) = 20$</td>
<td>$7(6) = 42$</td>
<td>$8(7) = 56$</td>
</tr>
</tbody>
</table>
Determining the Perimeter and Area of Triangles on the Coordinate Plane

The formula for the area of a triangle is half the area of a rectangle. Therefore, the area of a triangle can be found by taking half of the product of the base and the height. The height of a triangle must always be perpendicular to the base. On the coordinate plane, the slope of the height is the negative reciprocal of the slope of the base.

Example

Determine the perimeter and area of triangle $JDL$.

The vertices of triangle $JDL$ are $J(1, 6)$, $D(7, 9)$, and $L(8, 3)$.

$$JD = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(7 - 1)^2 + (9 - 6)^2} = \sqrt{6^2 + 3^2} = \sqrt{36 + 9} = \sqrt{45} = 3\sqrt{5}$$

$$DL = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(8 - 7)^2 + (3 - 9)^2} = \sqrt{1^2 + (-6)^2} = \sqrt{1 + 36} = \sqrt{37}$$

$$LJ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(1 - 8)^2 + (6 - 3)^2} = \sqrt{(-7)^2 + 3^2} = \sqrt{49 + 9} = \sqrt{58}$$

$$P = JD + DL + LJ = 3\sqrt{5} + \sqrt{37} + \sqrt{58} \approx 20.4$$

The perimeter of triangle $JDL$ is approximately 20.4 units.
To determine the area of the triangle, first determine the height of triangle $JDL$.

Slope of $\overline{JD}: m = \frac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{9 - 6}{7 - 1}$$

$$= \frac{3}{6}$$

$$= \frac{1}{2}$$

Slope of $\overline{PL}: m = -2$

Equation of $\overline{JD}: (y - y_1) = m(x - x_1)$

$$y - 6 = \frac{1}{2}(x - 1)$$

$$y = \frac{1}{2}x + \frac{5}{2}$$

Equation of $\overline{PL}: (y - y_1) = m(x - x_1)$

$$y - 3 = -2(x - 8)$$

$$y = -2x + 19$$

Intersection of $\overline{JD}$ and $\overline{PL}$, or $P$:

$$\frac{1}{2}x + 2x = 19 - \frac{5}{2}$$

$$\frac{3}{2}x = 13\frac{1}{2}$$

$$x = 5.4$$

$$y = -2(5.4) + 19$$

$$y = 8.2$$

The coordinates of $P$ are $(5.4, 8.2)$.

Height of triangle $JDL$: $PL = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$= \sqrt{(8 - 5.4)^2 + (3 - 8.2)^2}$$

$$= \sqrt{(2.6)^2 + (-5.2)^2}$$

$$= \sqrt{33.8}$$

$$\approx 5.8$$

Area of triangle $JDL$: $A = \frac{1}{2} bh$

$$= \frac{1}{2} JD(PL)$

$$= \frac{1}{2}(3\sqrt{5})(\sqrt{33.8})$$

$$= \frac{1}{2}(3\sqrt{169})$$

$$= 19.5$$

The area of triangle $JDL$ is 19.5 square units.
3.2 Doubling the Area of a Triangle

To double the area of a triangle, only the length of the base or the height of the triangle need to be doubled. If both the length of the base and the height are doubled, the area will quadruple.

Example

Double the area of triangle ABC by manipulating the height.

\[
\begin{align*}
\text{Area of } ABC & \quad \text{Area of } ABC' \\
A &= \frac{1}{2}bh & A &= \frac{1}{2}bh \\
&= \frac{1}{2}(5)(4) & &= \frac{1}{2}(5)(8) \\
&= 10 & &= 20
\end{align*}
\]

By doubling the height, the area of triangle \(ABC'\) is double the area of triangle \(ABC\).

3.3 Determining the Perimeter and Area of Parallelograms on the Coordinate Plane

The formula for calculating the area of a parallelogram is the same as the formula for calculating the area of a rectangle: \(A = bh\). The height of a parallelogram is the length of a perpendicular line segment from the base to a vertex opposite the base.

Example

Determine the perimeter and area of parallelogram WXYZ.
The vertices of parallelogram $WXYZ$ are $W(-3, -5)$, $X(3, -3)$, $Y(2, -5)$, and $Z(-4, -7)$.

$WX = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(3 - (-3))^2 + (-3 - (-5))^2}$

$= \sqrt{6^2 + 2^2}$

$= \sqrt{40}$

$= 2\sqrt{10}$

$YZ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-4 - 2)^2 + (-7 - (-5))^2}$

$= \sqrt{(-6)^2 + (-2)^2}$

$= \sqrt{40}$

$= 2\sqrt{10}$

$WZ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-4 - (-3))^2 + (-7 - (-5))^2}$

$= \sqrt{(-1)^2 + (-2)^2}$

$= \sqrt{5}$

$XY = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(2 - 3)^2 + (-5 - (-3))^2}$

$= \sqrt{(-1)^2 + (-2)^2}$

$= \sqrt{5}$

$P = WX + XY + YZ + WZ$

$= 3\sqrt{10} + \sqrt{5} + 2\sqrt{10} + \sqrt{5}$

$\approx 17.1$

The perimeter of parallelogram $WXYZ$ is approximately 17.1 units.

To determine the area of parallelogram $WXYZ$, first calculate the height, $AY$.

Slope of base $WX$: $m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-3 - (-5)}{3 - (-3)}$

$= \frac{2}{6}$

$= \frac{1}{3}$

Slope of height $AY$: $m = -3$

Equation of base $WX$: $(y - y_1) = m(x - x_1)$

$(y - (-3)) = \frac{1}{3}(x - 3)$

$y = \frac{1}{3}x - 4$

Equation of height $AY$: $(y - y_1) = m(x - x_1)$

$(y - (-5)) = -3(x - 2)$

$y = -3x + 1$

Intersection of $WX$ and $AY$, or $A$: $\frac{1}{3}x - 4 = -3x + 1$

$\frac{1}{3}x + 3x = 1 + 4$

$\frac{10}{3}x = 5$

$x = \frac{1}{2}$

$y = -3x + 1$

$y = -3(\frac{1}{2}) + 1$

$y = -3\frac{1}{2}$
The coordinates of point $A$ are $(1\frac{1}{2}, -2\frac{1}{2})$.

$$AY = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(2 - 1\frac{1}{2})^2 + (-5 - (-3\frac{1}{2}))^2}$$

$$= \sqrt{\left(\frac{1}{2}\right)^2 + \left(-1\frac{1}{2}\right)^2}$$

$$= \sqrt{2.5}$$

Area of parallelogram $WXYZ$: $A = bh$

$$A = 2\sqrt{10}(\sqrt{2.5})$$

$$A = 10$$

The area of parallelogram $WXYZ$ is 10 square units.

### 3.3 Doubling the Area of a Parallelogram

To double the area of a parallelogram, only the length of the bases or the height of the parallelogram needs to be doubled. If both the length of the bases and the height are doubled, the area will quadruple.

**Example**

Double the area of parallelogram $PQRS$ by manipulating the length of the bases.

Area of $PQRS$    Area of $PQR’S’$

$A = bh$    $A = bh$

$= (6)(3)$    $= (12)(3)$

$= 18$    $= 36$

By doubling the length of the bases, the area of parallelogram $PQR’S’$ is double the area of parallelogram $PQRS$. 
3.4 Determining the Perimeter and Area of Trapezoids on the Coordinate Plane

A trapezoid is a quadrilateral that has exactly one pair of parallel sides. The parallel sides are known as the bases of the trapezoid, and the non-parallel sides are called the legs of the trapezoid. The area of a trapezoid can be calculated by using the formula $A = \frac{b_1 + b_2}{2}h$, where $b_1$ and $b_2$ are the bases of the trapezoid and $h$ is a perpendicular segment that connects the two bases.

Example

Determine the perimeter and area of trapezoid GAME.

The coordinates of the vertices of trapezoid GAME are $G(-4, 18)$, $A(2, 12)$, $M(2, 0)$, and $E(-4, -6)$.

\[
\begin{align*}
GA &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
&= \sqrt{(2 - (-4))^2 + (12 - 18)^2} \\
&= \sqrt{6^2 + (-6)^2} \\
&= \sqrt{72} \\
&= 6\sqrt{2} \\
EG &= 18 - (-6) \\
&= 24 \\
AM &= 12 - 0 \\
&= 12 \\
P &= GA + AM + ME + EG \\
&= 6\sqrt{2} + 12 + 6\sqrt{2} + 24 \\
&\approx 53.0
\end{align*}
\]

The perimeter of trapezoid GAME is approximately 53.0 units.
3.5 Determining the Perimeter and Area of Composite Figures on the Coordinate Plane

A composite figure is a figure that is formed by combining different shapes. The area of a composite figure can be calculated by drawing line segments on the figure to divide it into familiar shapes and determining the total area of those shapes.

Example

Determine the perimeter and area of the composite figure.

The coordinates of the vertices of this composite figure are $P(-4, 9)$, $T(2, 6)$, $S(5, 6)$, $B(5, 1)$, $R(3, -5)$, $G(-2, -5)$, and $H(-4, 1)$.

$TS = 3$, $SB = 5$, $RG = 5$, $HP = 8$

\[
PT = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-4 - 2)^2 + (9 - 6)^2} = \sqrt{(-6)^2 + (3)^2} = \sqrt{36 + 9} = \sqrt{45} = 3\sqrt{5}
\]

\[
BR = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(5 - 2)^2 + (6 - 5)^2} = \sqrt{3 + 1} = \sqrt{4} = 2
\]

\[
RG = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(5 - 3)^2 + (6 - (-5))^2} = \sqrt{4 + 121} = \sqrt{125} = 5\sqrt{5}
\]

\[
GH = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(5 - (-2))^2 + (6 - (-5))^2} = \sqrt{49 + 121} = \sqrt{170}
\]

\[
HP = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-4 - 2)^2 + (1 - (-5))^2} = \sqrt{36 + 36} = \sqrt{72} = 6\sqrt{2}
\]

\[
P = PT + TS + SB + BR + RG + GH + HP = 3\sqrt{5} + 3 + 5 + 2\sqrt{10} + 5 + 2\sqrt{10} + 8 = 40.4
\]
The perimeter of the composite figure \( \text{PTSBRGH} \) is approximately 40.4 units.

The area of the figure is the sum of the triangle, rectangle, and trapezoid formed by the dotted lines.

<table>
<thead>
<tr>
<th>Area of triangle:</th>
<th>Area of rectangle:</th>
<th>Area of trapezoid:</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A = \frac{1}{2} bh )</td>
<td>( A = bh )</td>
<td>( A = \frac{1}{2} \left( \frac{b_1 + b_2}{2} \right) h )</td>
</tr>
<tr>
<td>( = \frac{1}{2} (6)(3) )</td>
<td>( = 9(5) )</td>
<td>( = \left( \frac{9 + 5}{2} \right) (6) )</td>
</tr>
<tr>
<td>( = 9 )</td>
<td>( = 45 )</td>
<td>( = 42 )</td>
</tr>
</tbody>
</table>

The area of composite figure: \( A = 9 + 45 + 42 \)
\( = 96 \)

The area of the composite figure \( \text{PTSBRGH} \) is 96 square units.